ASSIGNMENT V

a) Define Mathematical Modelling.

Answer

Mathematical modelling is defined as the act of describing a system and its process using mathematical concept and language. It is the process of setting up a model, solving it mathematically and interpreting the result in physical or other terms.

b) Outline two methods of obtaining mathematical model for engineering system.

Answer

(i) Transition from the physical situation (physical system) to its mathematical formulation (its mathematical model)

ii) Solution by a mathematical method.

c) A thermometer that initially reads $10^\circ c$ is used to measure the temperature of a system. The temperature of the thermometer is discovered to be $20^\circ c$ after 5mins of inserting it into the system. If the actual temperature of the system is $25^\circ c$.

i) Develop a model for the system.

ii) Simulate the developed model for time $(t) = 0$ to $t = 60$ mins using a step time of 1min with the aid of Microsoft excel.

iii) Obtain the dynamic response of the system with ~~which~~ the aid of MATLAB without using syms command, for $t = 0$ to $t = 60$min using a step of 1min.

iv) Using either the dynamic response write the steady state temperature of the system.

v) Using the developed model equation, evaluate the temperature of the thermometer as $t \to \infty$

Solution

i) Let $T(t)$ be the temperature of the system and T_A be the actual temperature, by Newton's law of cooling:

$$\frac{dT}{dt} = K(T - T_A)$$

Using seperation of variables

$$\frac{dT}{T - T_A} = k \cdot dt$$

Integrating both sides

$$\int \frac{dT}{T - T_A} = \int k\,dt$$

$$\ln(T - T_A) = kt + C$$

$$T - T_A = e^{kt + C}$$

$$T - T_A = e^{kt} \cdot e^{C}$$

$e^{c} = C$ . . . is the initial condition

$$T(t) = T_A + Ce^{kt}$$

To find C, giving the initial condition

$$T(0) = T_A + Ce^{k(0)}$$

$$10 = 25 + C$$

$$C = 10 - 25$$

$$C = -15$$

$$\therefore T(t) = 25 - 15e^{kt}$$

To find k

at $t = 5$mins, $T = 20^{\circ}c$

$$20 = 25 - 15e^{kt}$$

$$20 - 25 = -15e^{kt}$$

$$15e^{kt} = 5$$

$$e^{kt} = \frac{5}{15}$$

$$e^{kt} = \frac{1}{3}$$

$$kt = \ln(1/3)$$

$$5k = \ln(1/3)$$

$$k = \frac{\ln(1/3)}{5}$$

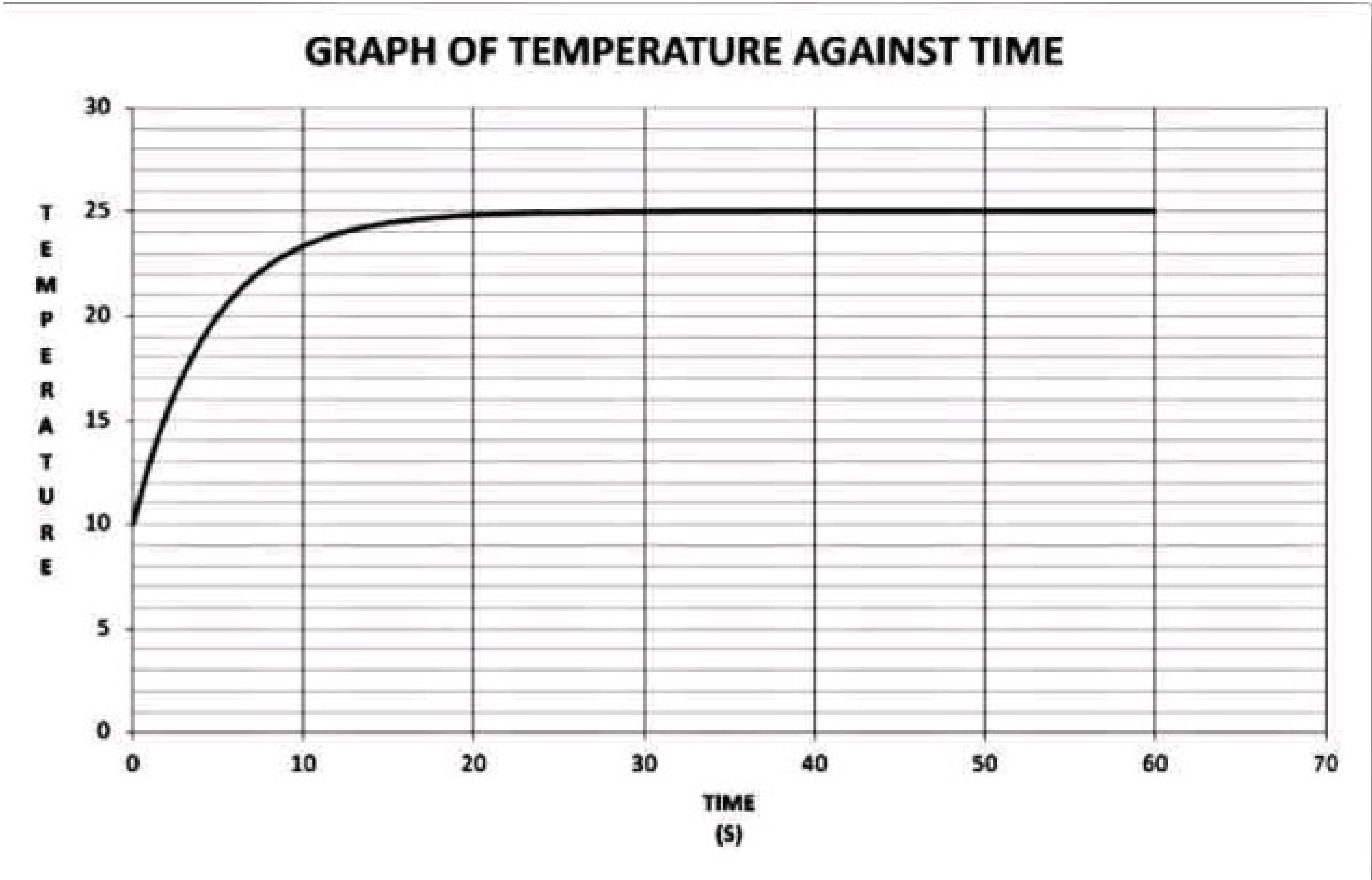
$$k = -0.22$$

$\therefore$

$$T = 25 - 15e^{-0.22t}$$

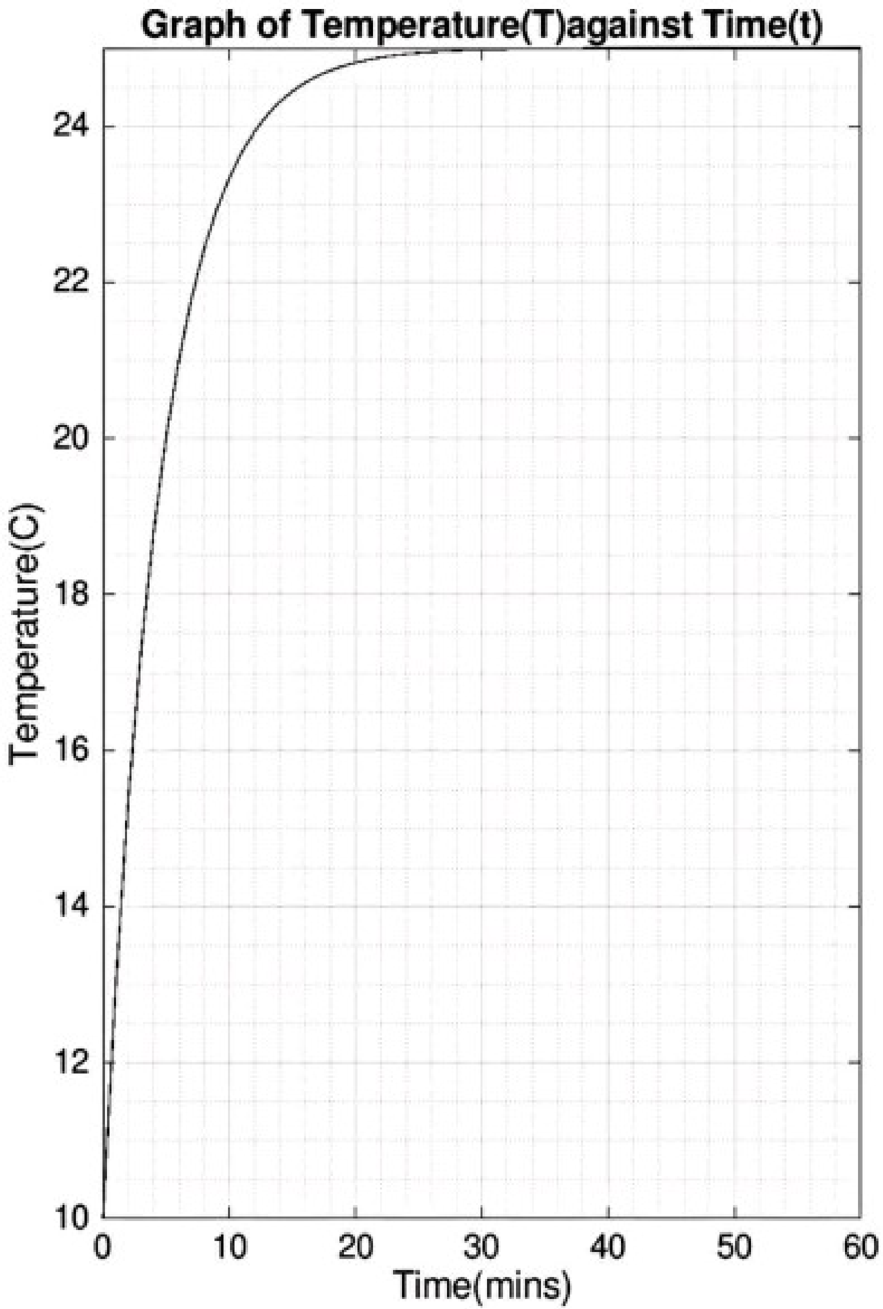
X(TIME)	Y(TEMPERATURE)=25-15*EXP(-0.22*A2)
0	10
1	12.96221803
2	15.33945368
3	17.24722998
4	18.77825632
5	20.00693374
6	20.99297047
7	21.78428348
8	22.41932704
9	22.92896144
10	23.33795262
11	23.66617574
12	23.92958096
13	24.1409686
14	24.31061115
15	24.44675249
16	24.55600847
17	24.64368845
18	24.71405329
19	24.77052239
20	24.8158399
21	24.85220806
22	24.88139419
23	24.90481661
24	24.92361354
25	24.93869843
26	24.95080434
27	24.96051956
28	24.9683162
29	24.97457316
30	24.97959448
31	24.98362419
32	24.9868581
33	24.98945338
34	24.99153614
35	24.99320759
36	24.99454897
37	24.99562544
38	24.99648933
39	24.99718263
40	24.997739
41	24.99818551
42	24.99854384
43	24.9988314

44	24.99906218
45	24.99924738
46	24.99939601
47	24.99951529
48	24.99961101
49	24.99968783
50	24.99974947
51	24.99979895
52	24.99983865
53	24.99987052
54	24.99989609
55	24.99991661
56	24.99993308
57	24.99994629
58	24.9999569
59	24.99996541
60	24.99997224


GRAPH OF TEMPERATURE AGAINST TIME
TEMPERATURE
30
25
20
15
10
5
0
0
10
20
30
40
50
60
70
TIME
(S)

iii) <u>CODES</u>

```
Commandwindow
Clear all
Clc
Close all
t = 0:1:60
T = 25-15*exp(-0.22*t)
Tn = Subs (T)
Plot (t, Tn)
xlabel('Time(s)')
ylabel('Temperature(°C)')
Title ('Graph of Temperature (T) against time(t)')
Grid On
Grid minor
```


Graph of Temperature(T)against Time(t)
Temperature(C)
Time(mins)
10
12
14
16
18
20
22
24
0
10
20
30
40
50
60

(iv) The steady state value of the temperature is 25°c at 30 mins of the exponential approach

(v) As t tends to infinity, the temperature approaches the steady state value which is 25°c.