

(a) Manual

Finite difference form

$$\frac{T_{i,j+1} - T_{i,j}}{\Delta t} = C \times \left[\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{(\Delta x)^2} \right]$$

where $C = 2.2 \text{ cm}^2/\text{hr}$

- making $T_{i,j+1} - T_{i,j}$ the subject of the formula

- subst. $C = 2.2 \text{ cm}^2/\text{hr}$

$$T_{i,j+1} - T_{i,j} = \frac{C \Delta t}{(\Delta x)^2} [T_{i+1,j} - 2T_{i,j} + T_{i-1,j}] \quad (*)$$

where $\frac{C \Delta t}{(\Delta x)^2} = r$

subst. $C = 2.2 \text{ cm}^2/\text{hr}$, $\Delta t = 0.02 \text{ hr}$, $\Delta x = 0.3 \text{ cm}$

$$r = \frac{2.2 \times (0.02)}{0.3^2} = 0.4889$$

$$r \approx 0.49$$

subst. $r = 0.49$ into equ (*)

$$\therefore T_{i,j+1} - T_{i,j} = 0.49 [T_{i+1,j} - 2T_{i,j} + T_{i-1,j}]$$

$$T_{i,j+1} = 0.49 T_{i+1,j} + 0.02 T_{i,j} + 0.49 T_{i-1,j}$$

for $t = 0 \text{ hr}$, $T(x, 0) = 3x^2 \rightarrow 1^{\text{st}} \text{ Row}$

where $x = 0 \text{ cm}$

$$T(0, 0) = x_0 = 3x(0)^2 = 0 \text{ cm}$$

where $x = 0.3 \text{ cm}$

$$T(0.3, 0) = x_1 = 3 \times (0.3)^2 = 0.27 \text{ cm}$$

where $x = 0.6 \text{ cm}$

$$T(0.6, 0) = x_2 = 3 \times (0.6)^2 = 1.08 \text{ cm}$$

where $x = 0.9 \text{ cm}$

$$T(0.9, 0) = x_3 = 3 \times (0.9)^2 = 2.4 \text{ cm}$$

where $x = 1.2 \text{ cm}$

$$T(1.2, 0) = x_4 = 3 \times (1.2)^2 = 4.32 \text{ cm}$$

where $x = 1.5 \text{ cm}$

$$T(1.5, 0) = x_5 = 3 \times (1.5)^2 = 6.75 \text{ cm}$$

where $x = 1.8 \text{ cm}$

$$T(1.8, 0) = x_6 = 3 \times (1.8)^2 = 9.72 \text{ cm}$$

where $x = 2.1 \text{ cm}$

$$T(2.1, 0) = x_7 = 3 \times (2.1)^2 = 13.23 \text{ cm}$$

where $x = 2.4 \text{ cm}$

$$T(2.4, 0) = T_8 = 3 \times (2.4)^2 = 17.28 \text{ cm}$$

where $x = 6.0 \text{ cm}$

$$T(6.0, 0) = T_{20} = 3(6.0)^2 = 108 \text{ cm}$$

2nd row, substitute.

$$T_{i,j+1} = 0.49 T_{i+1,j} + 0.02 T_{i,j} + 0.49 T_{i-1,j}$$

where $i=1, j=0$

$$\begin{aligned} T_{1,1} &= 0.49 T_{2,0} + 0.02 T_{1,0} + 0.49 T_{0,0} \\ &= 0.49(1.08) + 0.02(0.27) + 0.49(0) \end{aligned}$$

$$T_{1,1} = 0.5346$$

where $i=2, j=0$

$$\begin{aligned} T_{2,1} &= 0.49 T_{3,0} + 0.02 T_{2,0} + 0.49 T_{1,0} \\ &= 0.49(2.43) + 0.02(2.43) + 0.49(1.08) \end{aligned}$$

$$T_{2,1} = 1.8446$$

where $i=19, j=0$

$$\begin{aligned} T_{19,1} &= 0.49 T_{20,0} + 0.02 T_{19,0} + 0.49 T_{18,0} \\ &= 0.49(108) + 0.02(97.47) + 0.49(87.48) \end{aligned}$$

$$T_{19,1} = 97.7346$$

Table of values for up to $t = 0.02 \text{ s}$ and $x \leq 6 \text{ cm}$

DISTANCE (cm)

i =	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
x =	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3	3.3	3.6	3.9	4.2	4.5	4.8	5.1	5.4	5.7	6
j +	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}
0 0	0	0.2	1.08	2.43	4.32	6.75	9.72	13.23	17.28	21.87	27	32.67	38.61	45.63	52.92	60.48	68.28	76.38	84.78	93.48	102.48
1 0.02	0	0.5346	1.3440	2.6546	4.5546	7.0146	9.9846	13.4546	17.5446	22.1346	27.2046	32.7546	38.7946	45.3146	52.3146	59.7846	67.6846	75.9846	84.6846	93.7846	103.2846