

## ASSIGNMENT VII

1.) The model for the temperature distribution in a rod of length  $L = 6 \text{ cm}$  is as given in eqn 1.

$$\frac{dT}{dt}(x, t) = \frac{C d^2 T(x, t)}{dx^2}$$

where  $C = 2.2 \text{ cm}^2/\text{hr}$

with the conditions and the temperature ( $^{\circ}\text{C}$ )

$$T(x, 0) = 3x^2$$

$$T(0, t) = 0$$

$$T(L, t) = -108.$$

Using  $\Delta t = 0.02 \text{ hr}$  and  $\Delta x = 0.3 \text{ cm}$ , obtain the temperature profile of the system for  $0 \leq t \leq 0.3 \text{ hr}$ .

a.) Manually, in tabular form solving up to  $t = 0.02 \text{ hr}$  and  $x = 6 \text{ cm}$

Soln

Using Explicit forward Euler Method.

$$U_i^{k+1} = \gamma [U_i^k + U_{i+1}^k] + [1 - 2\gamma] U_i^k$$

when  $i = 1$

$$U_{1,j+1} = \gamma U_{2,j} + \gamma U_{0,j} + (1 - 2\gamma) U_{1,j}$$

$$\text{but } \gamma = \frac{C \cdot \Delta t}{(\Delta x)^2} = \frac{2.2 \times 0.02}{0.3^2} = 0.49$$

$$[1 - (2 \times 0.49)] = 0.02$$

Rewriting the explicit forward Euler method.

For when  $i = 1 \text{ to } 19$

$$U_{1,j+1} = 0.49 U_{0,j} + 0.49 U_{2,j} + 0.02 U_{1,j}$$

$$U_{2,j+1} = 0.49 U_{1,j} + 0.49 U_{3,j} + 0.02 U_{2,j}$$



$$\begin{aligned}
U_{3,j+1} &= 0.49U_{2,j} + 0.49U_{4,j} + 0.02U_{3,j} \\
U_{4,j+1} &= 0.49U_{3,j} + 0.49U_{5,j} + 0.02U_{4,j} \\
U_{5,j+1} &= 0.49U_{4,j} + 0.49U_{6,j} + 0.02U_{5,j} \\
U_{6,j+1} &= 0.49U_{5,j} + 0.49U_{7,j} + 0.02U_{6,j} \\
U_{7,j+1} &= 0.49U_{6,j} + 0.49U_{8,j} + 0.02U_{7,j} \\
U_{8,j+1} &= 0.49U_{7,j} + 0.49U_{9,j} + 0.02U_{8,j} \\
U_{9,j+1} &= 0.49U_{8,j} + 0.49U_{10,j} + 0.02U_{9,j} \\
U_{10,j+1} &= 0.49U_{9,j} + 0.49U_{11,j} + 0.02U_{10,j} \\
U_{11,j+1} &= 0.49U_{10,j} + 0.49U_{12,j} + 0.02U_{11,j} \\
U_{12,j+1} &= 0.49U_{11,j} + 0.49U_{13,j} + 0.02U_{12,j} \\
U_{13,j+1} &= 0.49U_{12,j} + 0.49U_{14,j} + 0.02U_{13,j} \\
U_{14,j+1} &= 0.49U_{13,j} + 0.49U_{15,j} + 0.02U_{14,j} \\
U_{15,j+1} &= 0.49U_{14,j} + 0.49U_{16,j} + 0.02U_{15,j} \\
U_{16,j+1} &= 0.49U_{15,j} + 0.49U_{17,j} + 0.02U_{16,j} \\
U_{17,j+1} &= 0.49U_{16,j} + 0.49U_{18,j} + 0.02U_{17,j} \\
U_{18,j+1} &= 0.49U_{17,j} + 0.49U_{19,j} + 0.02U_{18,j} \\
U_{19,j+1} &= 0.49U_{18,j} + 0.49U_{20,j} + 0.02U_{19,j}
\end{aligned}$$

For the boundary condition:

$T(x, 0) = 3x^2$  with  $x$  ranging from 0 to 6cm with step

Size of 0.3.

$$T(x_1, 0) = 3x^2 = 3(0.3)^2 = 0.27$$

$$T(x_2, 0) = 3x^2 = 3(0.6)^2 = 1.08$$

$$T(x_3, 0) = 3x^2 = 3(0.9)^2 = 2.43$$

$$T(x_4, 0) = 3x^2 = 3(1.2)^2 = 4.32$$

$$T(x_5, 0) = 3x^2 = 3(1.5)^2 = 6.75$$

$$T(x_6, 0) = 3x^2 = 3(1.8)^2 = 9.72$$

$$T(x_7, 0) = 3x^2 = 3(2.1)^2 = 13.23$$

$$T(x_8, 0) = 3x^2 = 3(2.4)^2 = 17.28$$

$$T(x_9, 0) = 3x^2 = 3(2.7)^2 = 21.87$$

$$T(x_{10}, 0) = 3x^2 = 3(3)^2 = 27$$

$$T(x_{11}, 0) = 3x^2 = 3(3.3)^2 = 32.67$$

$$T(x_{12}, 0) = 3x^2 = 3(3.6)^2 = 38.88$$

$$T(x_{13}, 0) = 3x^2 = 3(3.9)^2 = 45.63$$

$$T(x_{14}, 0) = 3x^2 = 3(4.2)^2 = 52.92$$

$$T(x_{15}, 0) = 3x^2 = 3(4.5)^2 = 60.75$$

$$T(x_{16}, 0) = 3x^2 = 3(4.8)^2 = 69.12$$

$$T(x_{17}, 0) = 3x^2 = 3(5.1)^2 = 78.03$$

$$T(x_{18}, 0) = 3x^2 = 3(5.4)^2 = 87.48$$

$$T(x_{19}, 0) = 3x^2 = 3(5.7)^2 = 97.03$$

$$T(x_{20}, 0) = 3x^2 = 3(6)^2 = 108$$



$$T(0,t) = 0, T(L,t) = 108.$$

temperature has a range of 0 to 0.3 hr with step size of 0.02 hr. To get to 0.02 hr,  $j = 0$

When  $j = 0$  [replacing  $U$  with  $T$ ]

$$\begin{aligned} T_{1,1} &= 0.49U_{0,0} + 0.49U_{2,0} + 0.02U_{1,0} \\ &= 0.49(0) + 0.49(1.08) + 0.02(0.27) \\ &= 0.5346 \end{aligned}$$

$$\begin{aligned} T_{2,1} &= 0.49U_{1,0} + 0.49U_{3,0} + 0.02U_{2,0} \\ &= 0.49(0.27) + 0.49(2.48) + 0.02(1.08) \\ &= 1.3446 \end{aligned}$$

$$\begin{aligned} T_{3,1} &= 0.49U_{2,0} + 0.49U_{4,0} + 0.02U_{3,0} \\ &= 0.49(1.08) + 0.49(4.32) + 0.02(2.43) \\ &= 2.6946 \end{aligned}$$

$$\begin{aligned} T_{4,1} &= 0.49U_{3,0} + 0.49U_{5,0} + 0.02U_{4,0} \\ &= 0.49[2.43] + 0.49(6.75) + 0.02[4.32] \\ &= 4.3846 \end{aligned}$$

$$\begin{aligned} T_{5,1} &= 0.49U_{4,0} + 0.49U_{6,0} + 0.02U_{5,0} \\ &= 0.49[4.32] + 0.49[9.72] + 0.02[6.75] \\ &= 7.0146 \end{aligned}$$

$$\begin{aligned} T_{6,1} &= 0.49T_{5,0} + 0.49T_{7,0} + 0.02T_{6,0} \\ &= 0.49[6.75] + 0.49[13.23] + 0.02[9.72] \\ &= 9.9846. \end{aligned}$$

$$\begin{aligned} T_{7,1} &= 0.49T_{6,0} + 0.49T_{8,0} + 0.02T_{7,0} \\ &= 0.49[9.72] + 0.49[17.25] + 0.02[13.23] \\ &= 13.4946 \end{aligned}$$

$$\begin{aligned} T_{8,1} &= 0.49T_{7,0} + 0.49T_{9,0} + 0.02T_{8,0} \\ &= 0.49(13.23) + 0.49[21.87] + 0.02[17.28] \\ &= 17.6446 \end{aligned}$$

$$\begin{aligned} T_{9,1} &= 0.49T_{8,0} + 0.49T_{10,0} + 0.02T_{9,0} \\ &= 0.49[17.28] + 0.49[22] + 0.02[21.87] \\ &= 22.1346 \end{aligned}$$



$$\begin{aligned}
 T_{10,1} &= 0.49 T_{9,0} + 0.49 T_{11,0} + 0.02 T_{10,0} \\
 &= 0.49 (21.87) + 0.49 [32.67] + 0.02 [27] \\
 &= 27.2646
 \end{aligned}$$

$$\begin{aligned}
 T_{11,1} &= 0.49 T_{10,0} + 0.49 T_{12,0} + 0.02 T_{11,0} \\
 &= 0.49 (27) + 0.49 (38.88) + 0.02 [32.67] \\
 &= 32.9346
 \end{aligned}$$

$$\begin{aligned}
 T_{12,1} &= 0.49 T_{11,0} + 0.49 T_{13,0} + 0.02 T_{12,0} \\
 &= 0.49 [32.67] + 0.49 [45.63] + 0.02 [38.88] \\
 &= 39.1446
 \end{aligned}$$

$$\begin{aligned}
 T_{13,1} &= 0.49 T_{12,0} + 0.49 T_{14,0} + 0.02 T_{13,0} \\
 &= 0.49 (38.88) + 0.49 (59.2) + 0.02 [45.63] = 45.8946
 \end{aligned}$$

$$\begin{aligned}
 T_{14,1} &= 0.49 T_{13,0} + 0.49 T_{15,0} + 0.02 T_{14,0} \\
 &= 0.49 (45.63) + 0.49 (60.75) + 0.02 [52.92] \\
 &= 53.1846
 \end{aligned}$$

$$\begin{aligned}
 T_{15,1} &= 0.49 T_{14,0} + 0.49 T_{16,0} + 0.02 T_{15,0} \\
 &= 0.49 (52.92) + 0.49 (69.12) + 0.02 [60.75] = 61.0146
 \end{aligned}$$

$$\begin{aligned}
 T_{16,1} &= 0.49 T_{15,0} + 0.49 T_{17,0} + 0.02 T_{16,0} \\
 &= 0.49 [60.75] + 0.49 [78.03] + 0.02 [69.12] \\
 &= 69.3846
 \end{aligned}$$

$$\begin{aligned}
 T_{17,1} &= 0.49 T_{16,0} + 0.49 T_{18,0} + 0.02 T_{17,0} \\
 &= 0.49 [69.12] + 0.49 [87.48] + 0.02 [78.03] \\
 &= 78.2946
 \end{aligned}$$

$$\begin{aligned}
 T_{18,1} &= 0.49 T_{17,0} + 0.49 T_{19,0} + 0.02 T_{18,0} \\
 &= 0.49 [78.03] + 0.49 [97.47] + 0.02 [87.48] \\
 &= 87.7446
 \end{aligned}$$

$$\begin{aligned}
 T_{19,1} &= 0.49 T_{18,0} + 0.49 T_{20,0} + 0.02 T_{19,0} \\
 &= 0.49 [87.48] + 0.49 [108] + 0.02 [97.47] \\
 &= 97.7346
 \end{aligned}$$