Oyeneyin Dapo Salm
16/ENG04/1050
Elect/Elect
ENG 382 Assignment VII

The model for the temperature distribution in a rod of length $L = 6cm$ is as given

$$\frac{dT(x,t)}{dt} = c\frac{dT(x,t)}{dt^2}$$

where $c = 0.0\ cm^2/hr$

With the conditions and the temperature (°C)

$T(a,0) = 30°$

$T(0,t) = 0$

$T(L,t) = -10°$

Using $\Delta t = 0.024$ and $\Delta x = 0.3cm$, obtain the temperature profile of the system for $0 \leq t \leq 0.3hr$

a) Manually, in tabular form serving up to $t = 0.07hr$ and $x = 6cm$

Soln

Using explicit forward euler method

$$u_i^{k+1} = r(u^k_{i+1} + u^k_{i-1}) + (1-2r)u_i^k$$

When $i = 1$

$$u_{i,i+1} = r u_{i,j} + r u_{0,j} + (1-2r) u_{i,j}$$

but $r = \frac{c\,dt}{(dx)^2} = 0.0 \times \frac{0.02}{0.3}$

$r = 0.49$

$r_1 = (0 \times 0.49) = 0.02$

Rewriting the explicit forward euler method

When $i = 1$ to 19

$u_{1,j+1} = 0.49u_{0,j} + 0.49u_{2,j} + 0.02u_{1,j}$

$u_{2,j+1} = 0.49u_{1,j} + 0.49u_{3,j} + 0.02u_{2,j}$

$u_{3,j+1} = 0.49u_{2,j} + 0.49u_{4,j} + 0.02u_{3,j}$

$V_{4,j+1} = 0.49V_{3,j} + 0.49V_{5,j} + 0.02V_{4,j}$

$V_{5,j+1} = 0.49V_{4,j} + 0.49V_{6,j} + 0.02V_{5,j}$

$V_{6,j+1} = 0.49V_{5,j} + 0.49V_{7,j} + 0.02V_{6,j}$

$V_{7,j+1} = 0.49V_{6,j} + 0.49V_{8,j} + 0.02V_{7,j}$

$V_{8,j+1} = 0.49V_{7,j} + 0.49V_{9,j} + 0.02V_{8,j}$

$V_{9,j+1} = 0.49V_{8,j} + 0.49V_{10,j} + 0.02V_{9,j}$

$V_{10,j+1} = 0.49V_{9,j} + 0.49V_{11,j} + 0.02V_{10,j}$

$V_{11,j+1} = 0.49V_{10,j} + 0.49V_{12,j} + 0.02V_{11,j}$

$V_{12,j+1} = 0.49V_{11,j} + 0.49V_{13,j} + 0.02V_{12,j}$

$V_{13,j+1} = 0.49V_{12,j} + 0.49V_{14,j} + 0.02V_{13,j}$

$V_{14,j+1} = 0.49V_{13,j} + 0.49V_{15,j} + 0.02V_{14,j}$

$V_{15,j+1} = 0.49V_{14,j} + 0.49V_{16,j} + 0.02V_{15,j}$

$V_{16,j+1} = 0.49V_{15,j} + 0.49V_{17,j} + 0.02V_{16,j}$

$V_{17,j+1} = 0.49V_{16,j} + 0.49V_{18,j} + 0.02V_{17,j}$

$V_{18,j+1} = 0.49V_{17,j} + 0.49V_{19,j} + 0.02V_{18,j}$

$V_{19,j+1} = 0.49V_{18,j} + 0.49V_{20,j} + 0.02V_{19,j}$

For the boundary condition

$T(x,0) = 3x^2$ with x running from 0 to 6cm with step size of 0.3

$T(x_1,0) = 3x^2 = 3 \times (0.3)^2 = 0.27$

$T(x_2,0) = 3(0.6)^2 = 1.08$

$T(x_3,0) = 3(0.9)^2 = 2.43$

$T(x_4,0) = 3(1.2)^2 = 4.32$

$T(x_5,0) = 3(1.5)^2 = 6.75$

$T(x_6,0) = 3(1.8)^2 = 9.72$

$T(x_7,0) = 3(2.1)^2 = 13.23$

$T(x_8,0) = 3(2.4)^2 = 17.28$

$T(x_9,0) = 3(2.7)^2 = 21.87$

$T(x_{10},0) = 3(3)^2 = 27$

$T(x_{11},0) = 3(3.3)^2 = 32.67$

$T(x_{12},0) = 3(3.6)^2 = 38.88$

$T(x_{13},0) = 3(3.9)^2 = 45.63$

$T(x_{14},0) = 3(4.2)^2 = 52.92$

$T(x_{15},0) = 3(4.5)^2 = 60.75$

$T(x_{16},0) = 3(4.8)^2 = 69.12$

$T(x_{2,0}) = 3(5.0)^2 = 78.03$

$T(x_{3,0}) = 3(5.4)^2 = 87.48$

$T(x_{4,0}) = 3(5.7)^2 = 94.1436$

$T(x_{20,0}) = 3(6)^2 = 108$

$T(0,t) = 0$, $T(L,t) = 108$

Temperature has a range of 0 to 0.3hr with step size of 0.02h, To get to 0.02hr $j = 0$

When $j = 0$ (relating U with T)

$T_{1,1} = 0.49u_{0,0} + 0.49u_{2,0} + 0.02u_{1,0}$
$= 0.49(0) + 0.49(1.08) + 0.02(0.22)$
$= 0.5346$

$T_{2,1} = 0.49u_{1,0} + 0.49u_{3,0} + 0.02u_{2,0}$
$= 0.49(0.22) + 0.49(2.43) + 0.02(1.08)$
$= 1.3446$

$T_{3,1} = 0.49u_{2,0} + 0.49u_{4,0} + 0.02u_{3,0}$
$= 0.49(1.08) + 0.49(4.32) + 0.02(2.43)$
$= 2.6941$

$T_{4,1} = 0.49u_{3,0} + 0.49u_{5,0} + 0.02u_{4,0}$
$= 0.49(2.43) + 0.49(6.75) + 0.02(4.32)$
$= 4.5846$

$T_{5,1} = 0.49u_{4,0} + 0.49u_{6,0} + 0.02u_{5,0}$
$= 0.49(4.32) + 0.49(9.72) + 0.02(6.75)$
$= 7.0146$

$T_{6,1} = 0.49u_{5,0} + 0.49u_{7,0} + 0.02u_{6,0}$
$= 0.49(6.75) + 0.49(13.23) + 0.02(9.72)$
$= 9.9846$

$T_{7,1} = 0.49u_{6,0} + 0.49u_{8,0} + 0.02u_{7,0}$
$= 0.49(9.72) + 0.49(17.28) + 0.02(13.23)$
$= 12.4946$

$T_{8,1} = 0.49u_{7,0} + 0.49u_{9,0} + 0.02u_{8,0}$
$= 0.49(13.33) + 0.49(21.87) + 0.02(17.28)$
$= 17.5446$

$T_{9,1} = 0.49u_{8,0} + 0.49u_{10,0} + 0.02u_{9,0}$
$= 0.49(17.28) + 0.49(27) + 0.02(21.87)$
$= 22.1346$

$T_{10,1} = 0.49U_{9,0} + 0.49U_{11,0} + 0.02U_{10,0}$
$= 0.49(21.87) + 0.49(32.67) + 0.02(27)$
$= 27.1646$

$T_{11,1} = 0.49U_{10,0} + 0.49U_{12,0} + 0.02U_{11,0}$
$= 0.49(87) + 0.49(88.88) + 0.02(33.62)$
$= 32.9346$

$T_{12,1} = 0.49U_{11,0} + 0.49U_{13,0} + 0.02U_{12,0}$
$= 0.49(32.67) + 0.49(45.63) + 0.02(38.36)$
$= 39.1446$

$T_{13,1} = 0.49U_{12,0} + 0.49U_{14,0} + 0.02U_{13,0}$
$= 0.49(38.88) + 0.49(59.2) + 0.02(45.68)$
$= 43.8946$

$T_{14,1} = 0.49U_{13,0} + 0.49U_{15,0} + 0.02U_{14,0}$
$= 0.49(43.43) + 0.49(40.28) + 0.02(52.92)$
$= 53.1846$

$T_{15,1} = 0.49U_{14,0} + 0.49U_{16,0} + 0.02U_{15,0}$
$= 0.49(32.92) + 0.49(69.12) + 0.02(60.75)$
$= 61.0146$

$T_{16,1} = 0.49U_{15,0} + 0.49U_{17,0} + 0.02U_{16,0}$
$= 0.49(60.75) + 0.49(78.03) + 0.02(69.17)$
$= 69.3846$

$T_{17,1} = 0.49U_{16,0} + 0.49U_{18,0} + 0.02U_{17,0}$
$= 0.49(69.12) + 0.49(87.48) + 0.02(78.08)$
$= 78.2946$

$T_{18,1} = 0.49U_{17,0} + 0.49U_{19,0} + 0.02U_{18,0}$
$= 0.49(78.03) + 0.49(97.47) + 0.02(87.48)$
$= 87.7446$

$T_{19,1} = 0.49U_{18,0} + 0.49U_{20,0} + 0.02U_{19,0}$
$= 0.49(87.48) + 0.49(108) + 0.02(97.42)$
$= 97.7346$