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Course: ENG 382

Assg 7

$$\frac{\partial T}{\partial t}(x,t) = C \frac{\partial^2 T}{\partial x^2}(x,t)$$

$$T(x,0) = 3x^2$$

$$T(0,t) = 0$$

$$T(L,t) = 108$$

$$C = 2.2 \text{ cm}^2/\text{hr}$$

Using explicit forward difference Euler method

$$\frac{1}{K} [U_{i,j+1} - U_{i,j}] = \frac{C}{h^2} [U_{i+1,j} + U_{i-1,j} - 2U_{i,j}]$$

$$\text{but } K = 0.02h$$

$$h = 0.3 \text{ cm}$$

$$r = \frac{K}{h^2} = \frac{0.02 \times 2.2}{0.3^2} = 0.49$$

$$U_{i,j+1} = rU_{i+1,j} + rU_{i-1,j} + (1-2r)U_{i,j}$$

per $i = 1$ to 19

$$U_{1,j+1} = 0.49U_{2,j} + 0.49U_{2,j} + 0.02U_{1,j}$$

$$U_{2,j+1} = 0.49U_{1,j} + 0.49U_{3,j} + 0.02U_{2,j}$$

$$U_{3,j+1} = 0.49U_{2,j} + 0.49U_{4,j} + 0.02U_{3,j}$$

$$U_{4,j+1} = 0.49U_{3,j} + 0.49U_{5,j} + 0.02U_{4,j}$$

$$U_{5,j+1} = 0.49U_{4,j} + 0.49U_{6,j} + 0.02U_{5,j}$$

$$U_{6,j+1} = 0.49U_{5,j} + 0.49U_{7,j} + 0.02U_{6,j}$$

$$U_{7,j+1} = 0.49U_{6,j} + 0.49U_{8,j} + 0.02U_{7,j}$$

$$U_{8,j+1} = 0.49U_{7,j} + 0.49U_{9,j} + 0.02U_{8,j}$$

$$U_{9,j+1} = 0.49U_{8,j} + 0.49U_{10,j} + 0.02U_{9,j}$$

$$U_{10,j+1} = 0.49U_{9,j} + 0.49U_{11,j} + 0.02U_{10,j}$$

$$u_{1,j+1} = 0.49 u_{0,j} + 0.49 u_{2,j} + 0.02 u_{1,j}$$

$$u_{2,j+1} = 0.49 u_{1,j} + 0.49 u_{3,j} + 0.02 u_{2,j}$$

$$u_{3,j+1} = 0.49 u_{2,j} + 0.49 u_{4,j} + 0.02 u_{3,j}$$

$$u_{4,j+1} = 0.49 u_{3,j} + 0.49 u_{5,j} + 0.02 u_{4,j}$$

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$$u_{9,j+1} = 0.49 u_{8,j} + 0.49 u_{9,j} + 0.02 u_{9,j}$$

for boundary condition,

$$T(x, 0) = 3x^2$$

$$T(x_1, 0) = 3x^2 = 3(0.3)^2 = 0.27$$

$$T(x_2, 0) = 3(0.6)^2 = 1.08$$

$$T(x_3, 0) = 3(0.9)^2 = 2.43$$

$$T(x_4, 0) = 3(1.2)^2 = 4.32$$

$$T(x_5, 0) = 3(1.5)^2 = 6.75$$

Replacing u with T ,

$$T_{1,1} = 0.49(0) + 0.49(1.08) + 0.02(0.27) = 0.5346$$

$$T_{2,1} = 0.49(0.27) + 0.49(2.43) + 0.02(1.08) = 1.3446$$

$$T_{3,1} = 0.49(1.08) + 0.49(4.32) + 0.02(2.43) = 2.64046$$

$$T_{4,1} = 0.49(2.43) + 0.49(6.75) + 0.02(4.32) = 4.5846$$

$$T_{5,1} = 0.49(4.32) + 0.49(9.72) + 0.02(6.75) = 7.0146$$

$$T_{6,1} = 0.49(6.75) + 0.49(13.23) + 0.02(9.72) = 9.9846$$

$$T_{7,1} = 0.49(9.72) + 0.49(17.28) + 0.02(13.23) = 13.4946$$

$$T_{8,1} = 0.49(13.23) + 0.49(21.87) + 0.02(17.28) = 17.5446$$

$$T_{9,1} = 0.49(17.28) + 0.49(27) + 0.02(21.87) = 22.1346$$

$$T_{10,1} = 0.49(21.87) + 0.49(32.67) + 0.02(27) = 27.2646$$

$$T_{11,1} = 0.49(27) + 0.49(38.88) + 0.02(32.67) = 32.9346$$

$$T_{12,1} = 0.49(32.67) + 0.49(45.63) + 0.02(38.88) = 39.1446$$

$$T_{13,1} = 0.49(38.88) + 0.49(52.92) + 0.02(45.63) = 45.8946$$

$$T_{14,1} = 0.49(45.63) + 0.49(60.75) + 0.02(52.92) = 53.1846$$

$$T_{15,j} = 0.49(52.92) + 0.49(69.12) + 0.02(60.75) = 61.0146$$

$$T_{16,j} = 0.49(60.75) + 0.49(78.03) + 0.02(69.12) = 69.3846$$

$$T_{17,j} = 0.49(69.12) + 0.49(87.48) + 0.02(78.03) = 78.2946$$

$$T_{18,j} = 0.49(78.03) + 0.49(97.47) + 0.02(87.48) = 87.7446$$

$$T_{19,j} = 0.49(87.48) + 0.49(108) + 0.02(97.47) = 97.7346$$

x	t		j
	j		
0.0	0	0	0
0.3	1	0.27	0.5346
0.6	2	1.08	1.3446
0.9	3	2.43	2.6446
1.2	4	4.32	4.5846
1.5	5	6.75	7.0146
1.8	6	9.75	9.9846
2.1	7	13.23	13.4946
2.4	8	17.28	17.5446
2.7	9	21.87	22.2346
3.0	10	27	27.2646
3.3	11	32.67	32.9346
3.6	12	38.88	39.1446
3.9	13	45.63	45.8946
4.2	14	52.92	53.1846
4.5	15	60.75	61.0146
4.8	16	69.12	69.3846
5.1	17	78.03	78.2946
5.4	18	87.48	87.7446
5.7	19	97.47	97.7346
6.0	20	108	108