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16/ENG05/023
Mechchanical Engineering
ENG 382

Assignment

$$\frac{\partial T}{\partial t}(x,t) = C\frac{\partial^2 T}{\partial x^2}$$

$T(x,0) = 3x^2$

$T(0,t) = 0$

$T(L,t) = 10t$

$C = 2.2\ cm^2/hr$

Using Explicit forward difference euler method

$$\frac{1}{k}[u_{i,j+1} - u_{i,j}] = \frac{c}{h^2}[u_{i+1,j} + u_{i-1,j} - 2u_{i,j}]$$

Let $k = 0.02h$

$h = 0.3m$

$$r = \frac{kc}{h^2} = \frac{0.02 \times 2.2}{0.3^2} = 0.49$$

$$u_{i,j+1} = ru_{i+1,j} + ru_{i-1,j} + (1-2r)u_{i,j}$$

for $i = 1$ to 19

$u_{1,j+1} = 0.49u_{0,j} + 0.49u_{2,j} + 0.02u_{1,j}$

$u_{2,j+1} = 0.49u_{1,j} + 0.49u_{3,j} + 0.02u_{2,j}$

$u_{3,j+1} = 0.49u_{2,j} + 0.49u_{4,j} + 0.02u_{3,j}$

$u_{4,j+1} = 0.49u_{3,j} + 0.49u_{5,j} + 0.02u_{4,j}$

$u_{5,j+1} = 0.49u_{4,j} + 0.49u_{6,j} + 0.02u_{5,j}$

$u_{6,j+1} = 0.49u_{5,j} + 0.49u_{7,j} + 0.02u_{6,j}$

$u_{7,j+1} = 0.49u_{6,j} + 0.49u_{8,j} + 0.02u_{7,j}$

$u_{8,j+1} = 0.49u_{7,j} + 0.49u_{9,j} + 0.02u_{8,j}$

$u_{9,j+1} = 0.49u_{8,j} + 0.49u_{10,j} + 0.02u_{9,j}$

$u_{10,j+1} = 0.49u_{9,j} + 0.49u_{11,j} + 0.02u_{10,j}$

$u_{11,j+1} = 0.49u_{10,j} + 0.49u_{12,j} + 0.02u_{11,j}$

$u_{12,j+1} = 0.49u_{11,j} + 0.49u_{13,j} + 0.02u_{12,j}$

$u_{13,j+1} = 0.49u_{12,j} + 0.49u_{14,j} + 0.02u_{13,j}$

$u_{4,j+1} = 0.49u_{3,j} + 0.49u_{4,j} + 0.02u_{4,j}$

$u_{5,j+1} = 0.49u_{4,j} + 0.49u_{5,j} + 0.02u_{5,j}$

$u_{6,j+1} = 0.49u_{5,j} + 0.49u_{6,j} + 0.02u_{6,j}$

$u_{7,j+1} = 0.49u_{6,j} + 0.49u_{7,j} + 0.02u_{7,j}$

$u_{8,j+1} = 0.49u_{7,j} + 0.49u_{8,j} + 0.02u_{8,j}$

$u_{9,j+1} = 0.49u_{8,j} + 0.49u_{9,j} + 0.02u_{9,j}$

for boundary condition

$T(x,0) = 3x^2$

$T(x,0) = 3x^2 = 3(0.3)^2 = 0.27$

$T(x_2,0) = 3(0.6)^2 = 1.08$

$T(x_3,0) = 3(0.9)^2 = 2.43$

$T(x_4,0) = 3(1.2)^2 = 4.32$

$T(x_5,0) = 3(1.5)^2 = 6.75$

Replacing u with T

$T_{1,1} = 0.49(0) + 0.49(1.08) + 0.02(0.27) = 0.5346$

$T_{2,1} = 0.49(0.27) + 0.49(2.43) + 0.02(1.08) = 1.3446$

$T_{3,1} = 0.49(1.08) + 0.49(4.32) + 0.02(2.43) = 2.6046$

$T_{4,1} = 0.49(2.43) + 0.49(6.75) + 0.02(4.32) = 4.5846$

$T_{5,1} = 0.49(4.32) + 0.49(9.72) + 0.02(6.75) = 7.0146$

$T_{6,1} = 0.49(6.75) + 0.49(13.23) + 0.02(9.72) = 9.9846$

$T_{7,1} = 0.49(13.23) + 0.49(17.28) + 0.02(13.23) = 13.4946$

$T_{8,1} = 0.49(13.23) + 0.49(27) + 0.02(21.87) = 22.1346$

$T_{9,1} = 0.49(21.87) + 0.49(32.67) + 0.02(27) = 27.2646$

$T_{11,1} = 0.49(27) + 0.49(38.88) + 0.02(72.67) = 32.9346$

$T_{12,1} = 0.49(32.67) + 0.49(45.63) + 0.02(38.88) = 39.1446$

$T_{13,1} = 0.49(38.88) + 0.49(52.92) + 0.02(45.63) = 45.8946$

$T_{14,1} = 0.49(45.63) + 0.49(60.75) + 0.02(52.92) = 53.1846$

$T_{15,1} = 0.49(52.92) + 0.49(69.12) + 0.02(65.25) = 61.0146$

$T_{16,1} = 0.49(60.75) + 0.49(78.03) + 0.02(69.12) = 69.3846$

$T_{17,1} = 0.49(69.12) + 0.49(87.48) + 0.02(78.03) = 78.2946$

$T_{18,1} = 0.49(78.03) + 0.49(97.22) + 0.02(87.48) = 87.7446$

$T_{19,1} = 0.49(87.48) + 0.49(108) + 0.02(97.47) = 97.7346$