Assignment VI

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16/ENG06/015
Mechanical Engineering
ENG 382

$$d = \alpha \beta^{t} \qquad \text{—— (1)}$$

Soln

(a) Eqn (1) is non-linear
finding log of both sides

$$\log d = \log \alpha + \log \beta^{t}$$

$$\log d = \log \alpha + t \log \beta$$

comparing to $y = mx + c$

$y = \log d$; $c = \log \alpha$; $x = t$; $m = \log \beta$

standard eqn : $y = a_0 + a_1 x$

$a_0 = c$; $a_1 = m$

S/N	x		y	xy	x^2	y^2
	t (hr)	d (m)	log d	x * y	x^2	y^2
1.	0	2	0.301029996	0	0	0.090619058
2.	1	5	0.698970004	0.698970004	1	0.488559067
3.	2	19	1.278753601	2.557507	4	1.635210772
4.	3	50	1.698970004	5.096910013	9	2.886499076
5.	4	151	2.178976947	8.715908	16	4.747940537
6.	5	470	2.672097858	13.36049	25	7.140106962
7.	6	1435	3.156851901	18.94111	36	9.965713925
8.	7	4512	3.654369091	25.58058	49	13.35441345
9.	8	12936	4.111800007	32.8944	64	16.9068993
10.	9	41125	4.614105911	41.52695	81	21.28997336
11.	10	111021	5.045405135	50.45405	100	25.45611297
Σ	55		29.41133046	199.826839	385	103.9620485

$$y = a_0 + a_1 x$$

$$\Sigma y = a_0 N + a_1 \Sigma x$$

$$\Sigma xy = a_0 \Sigma x + a_1 \Sigma x^2$$

from the table

$29.44193046 = a_0 \times 11 + 55a_1$ —— (iv)

$199.8268839 = 55a_0 + 385a_1$ —— (v)

Using elimination of unknowns

multiply eqn (iv) by 5 and (v) by 1

$\therefore 147.0566523 = 55a_0 + 275a_1$ —— (2iv)

$199.8268839 = 55a_0 + 385a_1$ —— (2v)

substracting eqn (2iv) from (2v)

we have, $52.77023167 = 110a_1$

$\therefore a_1 = 0.479729379$

multiply eqn (iv) by 7 and (v) by 1

$\therefore 205.8798132 = 77a_0 + 385a_1$ —— (3iv)

$199.8268839 = 55a_0 + 385a_1$ —— (3v)

substracting eqn (3v) from (3iv), we have:

$6.052729249 = 22a_0$

$\therefore a_0 = 0.27511042$

$\therefore \boxed{y = 0.27511042 + 0.479729379x}$

Recall

$a_0 = c = \log \alpha$

$\alpha = 10^{a_0}$

$\alpha = 10^{0.27511042}$

$\alpha = 1.884128072 \simeq 1.88$

$a_1 = m = \log \beta$

$\beta = 10^{0.479729379}$

$\beta = 3.018070489 \simeq 3.012$

$\therefore$ The estimated values of α & β are $\boxed{1.88}$ and $\boxed{3.02}$ respectively.

(d) (i) Correlation Coefficient, R

$$R = \frac{N\sum XY - (\sum X)(\sum Y)}{\sqrt{(N\sum X^2 - (\sum X)^2)(N\sum Y^2 - (\sum Y)^2)}}$$

$$R = \frac{(11 \times 199.8268839) - (55 \times 29.41133046)}{\sqrt{((11 \times 385) - (55)^2) \times ((11 \times 103.9820485) - (29.41133046)^2)}}$$

$$R = 0.999844832$$

(iii) Square of the Correlation Coefficient, R^2

$$R^2 = (0.999844832)^2$$

$$R^2 = 0.999689689$$

(c) The values for Correlation Coefficient obtained falls between the standard range for correlation, 0.8 – 1.0.

As $0.8 < 0.999844832 < 1.0$

Therefore, the variables x(t) and y (log cl) CORRELATE.

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EXCEL:

R29

t(hr)	d(m)	
0	2	0.693147
1	5	1.609438
2	19	2.944439
3	50	3.912023
4	151	5.01728
5	470	6.152733
6	1435	7.26892
7	4512	8.414496
8	12936	9.467769
9	41125	10.62437
10	111021	11.61747

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.999844832
R Square	0.999689689
Adjusted R Sq	0.99965521
Standard Erro	0.068038263
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	134.2198329	134.2198	28994.14193	4.22594E-17
Residual	9	0.041662847	0.004629		
Total	10	134.2614958			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.633465152	0.038378756	16.50562	4.90261E-08	0.546646374	0.72028393	0.546646374	0.720283931
X Variable 1	1.104617716	0.006487194	170.2767	4.22594E-17	1.089942664	1.11929277	1.089942664	1.119292768

MATLAB:

ENG382_ASSIGNMENT6.m

```
commandwindow
clear
clc
format long g
collins = xlsread('ENG382_ASSIGNMENT6_2')
t = collins(:,1)
d = collins(:,2)
d = log(d)
[xr xc] = size(t)
t0 = ones(xr,1)
t1 = [t0 t]
maxwell = regress(d,t1)
lnalpha = maxwell(1)
lnbeta = maxwell(2)
alpha = exp(maxwell(1))
beta = exp(maxwell(2))
lnd = lnalpha + lnbeta*t
Rvalue = corr(d,lnd)
Rsquare = Rvalue^2
```

```
beta =

         3.01807048903391


lnd =

        0.633465152361569
         1.73808286864186
         2.84270058492216
         3.94731830120246
         5.05193601748275
         6.15655373376305
         7.26117145004335
         8.36578916632364
         9.47040688260394
         10.5750245988842
         11.6796423151645


Rvalue =

         0.99984483235763


Rsquare =

        0.999689688792257

fx >>
```