

# Assignment VII

ARABIC UNIVERSITY ISRAEL

16161001010

Mechanical Engineering

ENG 382

$$\frac{\partial T(x,t)}{\partial t} = c \frac{\partial^2 T(x,t)}{\partial x^2}$$

Soln

⊙

Using explicit forward Euler method

$$U_i^{k+1} = rU_{i-1}^k + (1-2r)U_i^k + rU_{i+1}^k$$

$$r = c \frac{\Delta t}{(\Delta x)^2} = \frac{2.2 \times 0.02}{0.3^2}$$

$$r = 0.49$$

$$U_i^{k+1} = 0.49U_{i-1,k} + 0.02U_{i,k} + 0.49U_{i+1,k}$$

$i$

for  $i = 1$  to  $19$

1	$\rightarrow U_1^{k+1} = 0.49U_{0,k} + 0.02U_{1,k} + 0.49U_{2,k}$
2	$\rightarrow U_2^{k+1} = 0.49U_{1,k} + 0.02U_{2,k} + 0.49U_{3,k}$
3	$\rightarrow U_3^{k+1} = 0.49U_{2,k} + 0.02U_{3,k} + 0.49U_{4,k}$
4	$\rightarrow U_4^{k+1} = 0.49U_{3,k} + 0.02U_{4,k} + 0.49U_{5,k}$
5	$\rightarrow U_5^{k+1} = 0.49U_{4,k} + 0.02U_{5,k} + 0.49U_{6,k}$
6	$\rightarrow U_6^{k+1} = 0.49U_{5,k} + 0.02U_{6,k} + 0.49U_{7,k}$
7	$\rightarrow U_7^{k+1} = 0.49U_{6,k} + 0.02U_{7,k} + 0.49U_{8,k}$
8	$\rightarrow U_8^{k+1} = 0.49U_{7,k} + 0.02U_{8,k} + 0.49U_{9,k}$
9	$\rightarrow U_9^{k+1} = 0.49U_{8,k} + 0.02U_{9,k} + 0.49U_{10,k}$
10	$\rightarrow U_{10}^{k+1} = 0.49U_{9,k} + 0.02U_{10,k} + 0.49U_{11,k}$
11	$\rightarrow U_{11}^{k+1} = 0.49U_{10,k} + 0.02U_{11,k} + 0.49U_{12,k}$
12	$\rightarrow U_{12}^{k+1} = 0.49U_{11,k} + 0.02U_{12,k} + 0.49U_{13,k}$
13	$\rightarrow U_{13}^{k+1} = 0.49U_{12,k} + 0.02U_{13,k} + 0.49U_{14,k}$
14	$\rightarrow U_{14}^{k+1} = 0.49U_{13,k} + 0.02U_{14,k} + 0.49U_{15,k}$
15	$\rightarrow U_{15}^{k+1} = 0.49U_{14,k} + 0.02U_{15,k} + 0.49U_{16,k}$
16	$\rightarrow U_{16}^{k+1} = 0.49U_{15,k} + 0.02U_{16,k} + 0.49U_{17,k}$
17	$\rightarrow U_{17}^{k+1} = 0.49U_{16,k} + 0.02U_{17,k} + 0.49U_{18,k}$
18	$\rightarrow U_{18}^{k+1} = 0.49U_{17,k} + 0.02U_{18,k} + 0.49U_{19,k}$
19	$\rightarrow U_{19}^{k+1} = 0.49U_{18,k} + 0.02U_{19,k} + 0.49U_{20,k}$

## Boundary Conditions

$$T(x, 0) = 3x^2; \quad 0 \leq x \leq 6 \text{ cm}$$

$$T(x_1, 0) = 3(0.3)^2 = 0.27$$

$$T(x_2, 0) = 3(0.6)^2 = 1.08$$

$$T(x_3, 0) = 3(0.9)^2 = 2.43$$

$$T(x_4, 0) = 3(1.2)^2 = 4.32$$

$$T(x_5, 0) = 3(1.5)^2 = 6.75$$

$$T(x_6, 0) = 3(1.8)^2 = 9.72$$

$$T(x_7, 0) = 3(2.1)^2 = 13.23$$

$$T(x_8, 0) = 3(2.4)^2 = 17.28$$

$$T(x_9, 0) = 3(2.7)^2 = 21.87$$

$$T(x_{10}, 0) = 3(3)^2 = 27$$

$$T(x_{11}, 0) = 3(3.3)^2 = 32.67$$

$$T(x_{12}, 0) = 3(3.6)^2 = 38.88$$

$$T(x_{13}, 0) = 3(3.9)^2 = 45.63$$

$$T(x_{14}, 0) = 3(4.2)^2 = 52.92$$

$$T(x_{15}, 0) = 3(4.5)^2 = 60.75$$

$$T(x_{16}, 0) = 3(4.8)^2 = 69.12$$

$$T(x_{17}, 0) = 3(5.1)^2 = 78.03$$

$$T(x_{18}, 0) = 3(5.4)^2 = 87.48$$

$$T(x_{19}, 0) = 3(5.7)^2 = 97.47$$

$$T(x_{20}, 0) = 3(6)^2 = 108$$

$$T(0, t) = 0$$

$$T(L, t) = 108$$

Temperature has a range of 0 to 0.3 hr with step size of 0.02 hr

For 0.02 hr,  $k=0$

$$T_{1,1} = 0.49U_{0,0} + 0.49U_{2,0} + 0.02U_{1,0} \\ = 0.49(0) + 0.49(1.08) + 0.02(0.27) = 0.5346$$

$$T_{2,1} = 0.49U_{1,0} + 0.49U_{3,0} + 0.02U_{2,0} \\ = 0.49(0.27) + 0.49(2.43) + 0.02(1.08) = 1.3446$$

$$T_{3,1} = 0.49U_{2,0} + 0.49U_{4,0} + 0.02U_{3,0} \\ = 0.49(1.08) + 0.49(4.32) + 0.02(2.43) = 2.6946$$

$$T_{4,1} = 0.49U_{3,0} + 0.49U_{5,0} + 0.02U_{4,0} \\ = 0.49(2.43) + 0.49(6.75) + 0.02(4.32) = 4.5846$$

$$T_{5,1} = 0.49U_{4,0} + 0.49U_{6,0} + 0.02U_{5,0} \\ = 0.49(4.32) + 0.49(9.72) + 0.02(6.75) = 7.0146$$

$$T_{6,1} = 0.49U_{5,0} + 0.49U_{7,0} + 0.02U_{6,0} \\ = 0.49(6.75) + 0.49(13.23) + 0.02(9.72) = 9.9846$$

$$T_{7,1} = 0.49U_{6,0} + 0.49U_{8,0} + 0.02U_{7,0} \\ = 0.49(9.72) + 0.49(17.28) + 0.02(13.23) = 13.4946$$

$$T_{8,1} = 0.49U_{7,0} + 0.49U_{9,0} + 0.02U_{8,0} \\ = 0.49(13.23) + 0.49(21.87) + 0.02(17.28) = 17.5446$$





$$T_{9,1} = 0.49U_{8,0} + 0.49U_{10,0} + 0.02U_{9,0}$$

$$= 0.49(17.28) + 0.49(27) + 0.02(21.81) = \underline{22.1346}$$

$$T_{10,1} = 0.49U_{9,0} + 0.49U_{11,0} + 0.02U_{10,0}$$

$$= 0.49(21.81) + 0.49(32.67) + 0.02(27) = \underline{27.2646}$$

$$T_{11,1} = 0.49U_{10,0} + 0.49U_{12,0} + 0.02U_{11,0}$$

$$= 0.49(27) + 0.49(38.88) + 0.02(32.67) = \underline{32.9346}$$

$$T_{12,1} = 0.49U_{11,0} + 0.49U_{13,0} + 0.02U_{12,0}$$

$$= 0.49(32.67) + 0.49(45.63) + 0.02(38.88) = \underline{39.1446}$$

$$T_{13,1} = 0.49U_{12,0} + 0.49U_{14,0} + 0.02U_{13,0}$$

$$= 0.49(38.88) + 0.49(52.92) + 0.02(45.63) = \underline{45.8946}$$

$$T_{14,1} = 0.49U_{13,0} + 0.49U_{15,0} + 0.02U_{14,0}$$

$$= 0.49(45.63) + 0.49(60.75) + 0.02(52.92) = \underline{53.1846}$$

$$T_{15,1} = 0.49U_{14,0} + 0.49U_{16,0} + 0.02U_{15,0}$$

$$= 0.49(52.92) + 0.49(69.12) + 0.02(60.75) = \underline{61.0146}$$

$$T_{16,1} = 0.49U_{15,0} + 0.49U_{17,0} + 0.02U_{16,0}$$

$$= 0.49(60.75) + 0.49(78.03) + 0.02(69.12) = \underline{69.3846}$$

$$T_{17,1} = 0.49U_{16,0} + 0.49U_{18,0} + 0.02U_{17,0}$$

$$= 0.49(69.12) + 0.49(87.48) + 0.02(78.03) = \underline{78.2946}$$

$$T_{18,1} = 0.49U_{17,0} + 0.49U_{19,0} + 0.02U_{18,0}$$

$$= 0.49(78.03) + 0.49(97.47) + 0.02(87.48) = \underline{87.7446}$$

$$T_{19,1} = 0.49U_{18,0} + 0.49U_{20,0} + 0.02U_{19,0}$$

$$= 0.49(87.48) + 0.49(108) + 0.02(97.47) = \underline{97.7346}$$

Table for Solving up to  $t = 0.02 \text{ hr}$  &  $x = 6 \text{ cm}$

$x$	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0	3.3
$T(x)$	0	0.27	1.08	2.43	4.32	6.75	9.72	13.23	17.28	21.81	27.00	32.67
0.02	0	0.5346	1.3446	2.6746	4.5846	7.0146	9.9846	13.4946	17.5146	22.1346	27.2646	32.9346

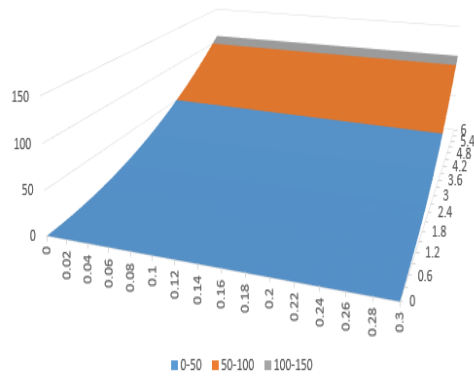
3.6	3.9	4.2	4.5	4.8	5.1	5.4	5.7	6.0
38.88	45.63	52.92	60.75	69.12	78.03	87.48	97.47	108
39.1446	45.8946	53.1846	61.0146	69.3846	78.2946	87.7446	97.7346	108

# EXCEL:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
	x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
t		0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3	3.3	3.6	3.9	4.2	4.5	4.8	5.1	5.4	5.7	6	
0	0	0	0.27	1.08	2.43	4.32	6.75	9.72	13.23	17.28	21.87	27	32.67	38.88	45.63	52.92	60.75	69.12	78.03	87.48	97.47	108	
1	0.02	0	0.534	1.344	2.694	4.584	7.014	9.984	13.494	17.544	22.134	27.264	32.934	39.144	45.894	53.184	61.014	69.384	78.294	87.744	97.734	108	
2	0.04	0	0.668933	1.608	2.958	4.848	7.278	10.248	13.758	17.808	22.398	27.528	33.198	39.408	46.158	53.448	61.278	69.648	78.558	88.008	97.86893	108	
3	0.06	0	0.800999	1.808901	3.222	5.112	7.542	10.512	14.022	18.072	22.662	27.792	33.462	39.672	46.422	53.712	61.542	69.912	78.822	88.2089	98.001	108	
4	0.08	0	0.902151	2.006997	3.455151	5.376	7.806	10.776	14.286	18.336	22.926	28.056	33.726	39.936	46.686	53.976	61.806	70.176	79.05515	88.407	98.10215	108	
5	0.1	0	1.001246	2.174837	3.686246	5.624918	8.07	11.04	14.55	18.6	23.19	28.32	33.99	40.2	46.95	54.24	62.07	70.42492	79.28625	88.57484	98.20125	108	
6	0.12	0	1.085504	2.339993	3.89513	5.872496	8.326627	11.304	14.814	18.864	23.454	28.584	34.254	40.464	47.214	54.504	62.32663	70.6725	79.49513	88.73999	98.2855	108	
7	0.14	0	1.168119	2.486976	4.101553	6.105581	8.582434	11.5644	15.078	19.128	23.718	28.848	34.518	40.728	47.478	54.7644	62.58243	70.90558	79.70155	88.88698	98.36812	108	
8	0.16	0	1.241813	2.63155	4.291952	6.33674	8.829376	11.82431	15.34024	19.392	23.982	29.112	34.782	40.992	47.74024	55.02431	62.82938	71.13674	79.89195	89.03155	98.44181	108	
9	0.18	0	1.314132	2.763875	4.479874	6.555688	9.074944	12.07902	15.6022	19.65514	24.246	29.376	35.046	41.25514	48.0022	55.27902	63.07494	71.35569	80.07987	89.16388	98.51413	108	
10	0.2	0	1.380431	2.894044	4.655783	6.772482	9.311966	12.3328	15.86119	19.91812	24.50958	29.64	35.30958	41.51812	48.26119	55.5328	63.31197	71.57248	80.25578	89.29404	98.58043	108	
11	0.22	0	1.445542	3.01535	4.829319	6.979177	9.547295	12.58094	16.11959	20.17945	24.77307	29.90359	35.57307	41.77945	48.51959	55.78094	63.54729	71.77918	80.42932	89.41535	98.64554	108	
12	0.24	0	1.506294	3.134718	4.993532	7.18366	9.774886	12.82783	16.3744	20.4404	25.03555	30.16708	35.83555	42.0404	48.7744	56.02783	63.77489	71.98366	80.59353	89.53472	98.70629	108	
13	0.26	0	1.566002	3.247353	5.155508	7.379752	10.00062	13.06916	16.62835	20.6991	25.29778	30.42959	36.09778	42.2991	49.02835	56.26916	64.00062	72.17975	80.75551	89.64735	98.766	108	
14	0.28	0	1.622395	3.358235	5.31004	7.573655	10.21948	13.30903	16.87844	20.9572	25.55842	30.69182	36.35842	42.5572	49.27844	56.50903	64.21948	72.37365	80.91004	89.75823	98.82239	108	
15	0.3	0	1.677857	3.463818	5.46248	7.760514	10.43641	13.54363	17.12745	21.21263	25.8186	30.95249	36.6186	42.81263	49.52745	56.74363	64.43641	72.56051	81.06248	89.86382	98.87786	108	

0.488889

### 3D graphical forms.



## MATLAB:

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Editor - C:\Users\ADMIN\Desktop\PRESENT SHII TO BE WORKED ON\ass.7\ENG382_ASSIGNMENT7.m
ENG382_ASSIGNMENT6.m x ENG382_ASSIGNMENT7.m x +
1 - commandwindow
2 - clear
3 - clc
4 - close all
5 - format short g
6 - tf = 0.3;
7 - lf = 6;
8 - nl = 21;
9 - nt = 16;
10 - dl = 0.3;
11 - dt = 0.02;
12 - r = 2.2*(dt/(dl^2));
13 - t = 0:dt:tf;
14 - l = [0:dl:lf]';
15 - T(1,1:length(t)) = 0;
16 - T(length(l),1:length(l)) = 108;
17 - T(1:length(l),1) = 3*(l.^2);
18 - for j = 1:nl
19 -     for i = 2:nt
20 -         T(j+1,i) = r*T(j,i-1) + (1-(2*r))*T(j,i) + r*T(j,1+1);
21 -     end
22 - end
23 - T
24 - % T(2,2) = r*T(1,1) + (1-2*r)*T(2,1) + r*T(3,1)
25 - % T(3,2) = r*T(2,1) + (1-2*r)*T(3,1) + r*T(4,1)
26 - % T(4,2) = r*T(3,1) + (1-2*r)*T(4,1) + r*T(5,1)
27 - % T(5,2) = r*T(4,1) + (1-2*r)*T(5,1) + r*T(6,1)
28 - % T(6,2) = r*T(5,1) + (1-2*r)*T(6,1) + r*T(7,1)
29 - % T(7,2) = r*T(6,1) + (1-2*r)*T(7,1) + r*T(8,1)
30 - % T(8,2) = r*T(7,1) + (1-2*r)*T(8,1) + r*T(9,1)
31 - % T(9,2) = r*T(8,1) + (1-2*r)*T(9,1) + r*T(10,1)
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Editor - C:\Users\ADMIN\Desktop\PRESENT SHII TO BE WORKED ON\ass.7\ENG382_ASSIGNMENT7.m
ENG382_ASSIGNMENT6.m ENG382_ASSIGNMENT7.m +
30 % T(8,2)= r*T(7,1) + (1-2*r)*T(8,1) + r*T(9,1)
31 % T(9,2)= r*T(8,1) + (1-2*r)*T(9,1) + r*T(10,1)
32 % T(10,2)= r*T(9,1) + (1-2*r)*T(10,1) + r*T(11,1)
33 % T(11,2)= r*T(10,1) + (1-2*r)*T(11,1) + r*T(12,1)
34 % T(12,2)= r*T(11,1) + (1-2*r)*T(12,1) + r*T(13,1)
35 % T(13,2)= r*T(12,1) + (1-2*r)*T(13,1) + r*T(14,1)
36 % T(14,2)= r*T(13,1) + (1-2*r)*T(14,1) + r*T(15,1)
37 % T(15,2)= r*T(14,1) + (1-2*r)*T(15,1) + r*T(16,1)
38 % T(16,2)= r*T(15,1) + (1-2*r)*T(16,1) + r*T(17,1)
39 % T(17,2)= r*T(16,1) + (1-2*r)*T(17,1) + r*T(18,1)
40 % T(18,2)= r*T(17,1) + (1-2*r)*T(18,1) + r*T(19,1)
41 % T(19,2)= r*T(18,1) + (1-2*r)*T(19,1) + r*T(20,1)
42 % T(20,2)= r*T(19,1) + (1-2*r)*T(20,1) + r*T(21,1)
43 %
44 % T(2,3)= r*T(1,2) + (1-2*r)*T(2,2) + r*T(3,2)
45 % T(3,3)= r*T(2,2) + (1-2*r)*T(3,2) + r*T(4,2)
46 % T(4,3)= r*T(3,2) + (1-2*r)*T(4,2) + r*T(5,2)
47 % T(5,3)= r*T(4,2) + (1-2*r)*T(5,2) + r*T(6,2)
48 % T(6,3)= r*T(5,2) + (1-2*r)*T(6,2) + r*T(7,2)
49 % T(7,3)= r*T(6,2) + (1-2*r)*T(7,2) + r*T(8,2)
50 % T(8,3)= r*T(7,2) + (1-2*r)*T(8,2) + r*T(9,2)
51 % T(9,3)= r*T(8,2) + (1-2*r)*T(9,2) + r*T(10,2)
52 % T(10,3)= r*T(9,2) + (1-2*r)*T(10,2) + r*T(11,2)
53 % T(11,3)= r*T(10,2) + (1-2*r)*T(11,2) + r*T(12,2)
54 % T(12,3)= r*T(11,2) + (1-2*r)*T(12,2) + r*T(13,2)
55 % T(13,3)= r*T(12,2) + (1-2*r)*T(13,2) + r*T(14,2)
56 % T(14,3)= r*T(13,2) + (1-2*r)*T(14,2) + r*T(15,2)
57 % T(15,3)= r*T(14,2) + (1-2*r)*T(15,2) + r*T(16,2)
58 % T(16,3)= r*T(15,2) + (1-2*r)*T(16,2) + r*T(17,2)
59 % T(17,3)= r*T(16,2) + (1-2*r)*T(17,2) + r*T(18,2)
60 % T(18,3)= r*T(17,2) + (1-2*r)*T(18,2) + r*T(19,2)
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