

Assignment 7

$$\frac{\partial T(x,t)}{\partial t} = c \frac{\partial^2 T(x,t)}{\partial x^2}$$

$$T(x,0) = 3x^2$$

$$T(0,t) = 0$$

$$T(L,t) = 108$$

$$c = \frac{2.2 \text{ cm}^2}{\text{hr}}$$

Using Explicit Forward Euler method.

$$\frac{1}{h} [U_{i,j+1} - U_{i,j}] = \frac{c}{h^2} [U_{i+1,j} + U_{i-1,j} - 2U_{i,j}]$$

$$r = \frac{kc}{h^2} =$$

$$k = 0.02h \quad ; h = 0.3 \text{ cm}$$

$$r = \frac{0.02 \times 2.2}{0.3^2} = 0.49$$

$$U_{i,j+1} = rU_{i+1,j} + rU_{i-1,j} + (1-2r)U_{i,j}$$

For $i = 1$ to 19

$$U_{1,j+1} = 0.49U_{0,j} + 0.49U_{2,j} + 0.02U_{1,j}$$

$$U_{2,j+1} = 0.49U_{1,j} + 0.49U_{3,j} + 0.02U_{2,j}$$

$$U_{3,j+1} = 0.49U_{2,j} + 0.49U_{4,j} + 0.02U_{3,j}$$

$$U_{4,j+1} = 0.49U_{3,j} + 0.49U_{5,j} + 0.02U_{4,j}$$

$$U_{5,j+1} = 0.49U_{4,j} + 0.49U_{6,j} + 0.02U_{5,j}$$

$$U_{6,j+1} = 0.49 U_{5,j} + 0.49 U_{7,j} + 0.02 U_{6,j}$$

$$U_{7,j+1} = 0.49 U_{6,j} + 0.49 U_{8,j} + 0.02 U_{7,j}$$

$$U_{8,j+1} = 0.49 U_{7,j} + 0.49 U_{9,j} + 0.02 U_{8,j}$$

$$U_{9,j+1} = 0.49 U_{8,j} + 0.49 U_{10,j} + 0.02 U_{9,j}$$

$$U_{10,j+1} = 0.49 U_{9,j} + 0.49 U_{11,j} + 0.02 U_{10,j}$$

$$U_{11,j+1} = 0.49 U_{10,j} + 0.49 U_{12,j} + 0.02 U_{11,j}$$

$$U_{12,j+1} = 0.49 U_{11,j} + 0.49 U_{13,j} + 0.02 U_{12,j}$$

$$U_{13,j+1} = 0.49 U_{12,j} + 0.49 U_{14,j} + 0.02 U_{13,j}$$

$$U_{14,j+1} = 0.49 U_{13,j} + 0.49 U_{15,j} + 0.02 U_{14,j}$$

$$U_{15,j+1} = 0.49 U_{14,j} + 0.49 U_{16,j} + 0.02 U_{15,j}$$

$$U_{16,j+1} = 0.49 U_{15,j} + 0.49 U_{17,j} + 0.02 U_{16,j}$$

$$U_{17,j+1} = 0.49 U_{16,j} + 0.49 U_{18,j} + 0.02 U_{17,j}$$

$$U_{18,j+1} = 0.49 U_{17,j} + 0.49 U_{19,j} + 0.02 U_{18,j}$$

$$U_{19,j+1} = 0.49 U_{18,j} + 0.49 U_{20,j} + 0.02 U_{19,j}$$

for boundary condition

$$T(x,0) = 3x^2$$

$$T(x_0,0) = 3x^2 = 3 \times 0.3^2 = 0.27$$

$$T(x_1,0) = 3 \times 0.6^2 = 1.08$$

$$T(x_2,0) = 3 \times 0.9^2 = 2.43$$

$$T(x_3,0) = 3 \times 1.2^2 = 4.32$$

$$T(x_4,0) = 3 \times 1.5^2 = 6.75$$

Replacing U with T

$$T_{1,1} = 0.49(0) + 0.49(1.08) + 0.02(0.27) = 0.5346$$

$$T_{2,1} = 0.49(0.27) + 0.49(2.43) + 0.02(1.08) = 1.3446$$

$$T_{3,1} = 0.49(1.08) + 0.49(4.32) + 0.02(2.43) = 2.64946$$

$$T_{4,1} = 0.49(2.43) + 0.49(6.75) + 0.02(4.32) = 4.5846$$

$$T_{5,1} = 0.49(4.32) + 0.49(9.72) + 0.02(6.75) = 7.0146$$

$$T_{6,1} = 0.49(6.75) + 0.49(13.23) + 0.02(9.72) = 9.9846$$

$$T_{7,1} = 0.49(9.72) + 0.49(17.28) + 0.02(13.23) = 13.4946$$

$$\begin{aligned}
T_{8,1} &= 0.49(13.23) + 0.49(21.87) + 0.02(17.28) = 17.5446 \\
T_{9,1} &= 0.49(17.28) + 0.49(22) + 0.02(21.87) = 22.1346 \\
T_{10,1} &= 0.49(21.87) + 0.49(32.67) + 0.02(21.87) = 27.2646 \\
T_{11,1} &= 0.49(\cancel{22.67}) + 0.49(38.88) + 0.02(32.67) = 32.9346 \\
T_{12,1} &= 0.49(32.67) + 0.49(45.63) + 0.02(38.88) = 39.1446 \\
T_{13,1} &= 0.49(38.88) + 0.49(59.2) + 0.02(45.63) = 45.8946 \\
T_{14,1} &= 0.49(45.63) + 0.49(60.75) + 0.02(52.92) = 53.1846 \\
T_{15,1} &= 0.49(52.92) + 0.49(69.12) + 0.02(60.75) = 61.0146 \\
T_{16,1} &= 0.49(60.75) + 0.49(78.03) + 0.02(69.18) = 69.3846 \\
T_{17,1} &= 0.49(69.12) + 0.49(87.48) + 0.02(78.03) = 78.2946 \\
T_{18,1} &= 0.49(78.03) + 0.49(97.47) + 0.02(87.48) = 87.7446 \\
T_{19,1} &= 0.49(87.48) + 0.49(108) + 0.02(97.47) = 97.7346
\end{aligned}$$

