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Matric Number: 11/mhs01/187

Assignment 1

1 The dynamic model of a body in motion performing damped force vibration is an equation (1)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that when $t=0$, $x = 0.1$ and $\frac{dx}{dt} = 0$.

(a) Using the auxiliary equation method, obtain the solution of the model in form of an expression having x as a function of t.

(b) With aid of a MATLAB mfile program plot the relationship between x and t for $0 \le t \le 15$ time unit using a step size of 0.01 unit and

(c) write the steady state solution of the model in form of $x = k\sin(t+a)$

Solution:

(a) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

Auxillary Equation: $m^2 + 5m + 6 = 0$

$m^2 + 2m + 3m + 6 = 0$

$m(m+2) + 3(m+2) = 0$

$(m+3)(m+2) = 0$

$m = -3$ or -2

Complementary function: $x = Ae^{-3x} + Be^{-2x}$

Particular Integral: Assume $x = C\cos t + D\sin t$

$\therefore \frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

Putting the value of d^2x/dt^2 & dx/dt in the equation

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

Comparing coefficient

Coefficient of cost : $-C+5D+6C = 1$ ---------- (1)

Coefficient of sint : $-D-5C+6D = 0$ ---------- (2)

from eqn (1) $5C+5D = 1$ ---------- (3)

from eqn (2) $5D-5C = 0$ ---------- (4)

from eqn (1) $5D = 1-5C$ ---------- (5)

put eqn (5) in eqn (4)

$$1-5C-5C=0$$

$$1-10C=0$$

$$C = 1/10 \; ; \; 0.1$$

$\therefore 5D = 1-5C$

Put the value of C in eqn (5)

$$5D = 1-5(0.1)$$

$$5D = 0.5$$

$$D = \frac{0.5}{5} = 0.1$$

Particular solution: $x = Ae^{-3t} + Be^{-2t} + 0.1\sin t + 0.1\cos t$

when $t=0$; $x = 0.1$, $\frac{dx}{dt} = 0$

$$0.1 = A+B+0.1$$

$A+B = 0$ ---------- (6)

$$\frac{dy}{dx} = -3Ae^{-3t} - 2Be^{-2t} + 0.1\sin t + 0.1\cos t$$

$$0 = -3A-2B+0.1$$

from eqn (6) $B = -A$

$$0 = -3A-2(-A)+0.1$$

$$0 = -A+0.1$$

$$A = 0.1$$

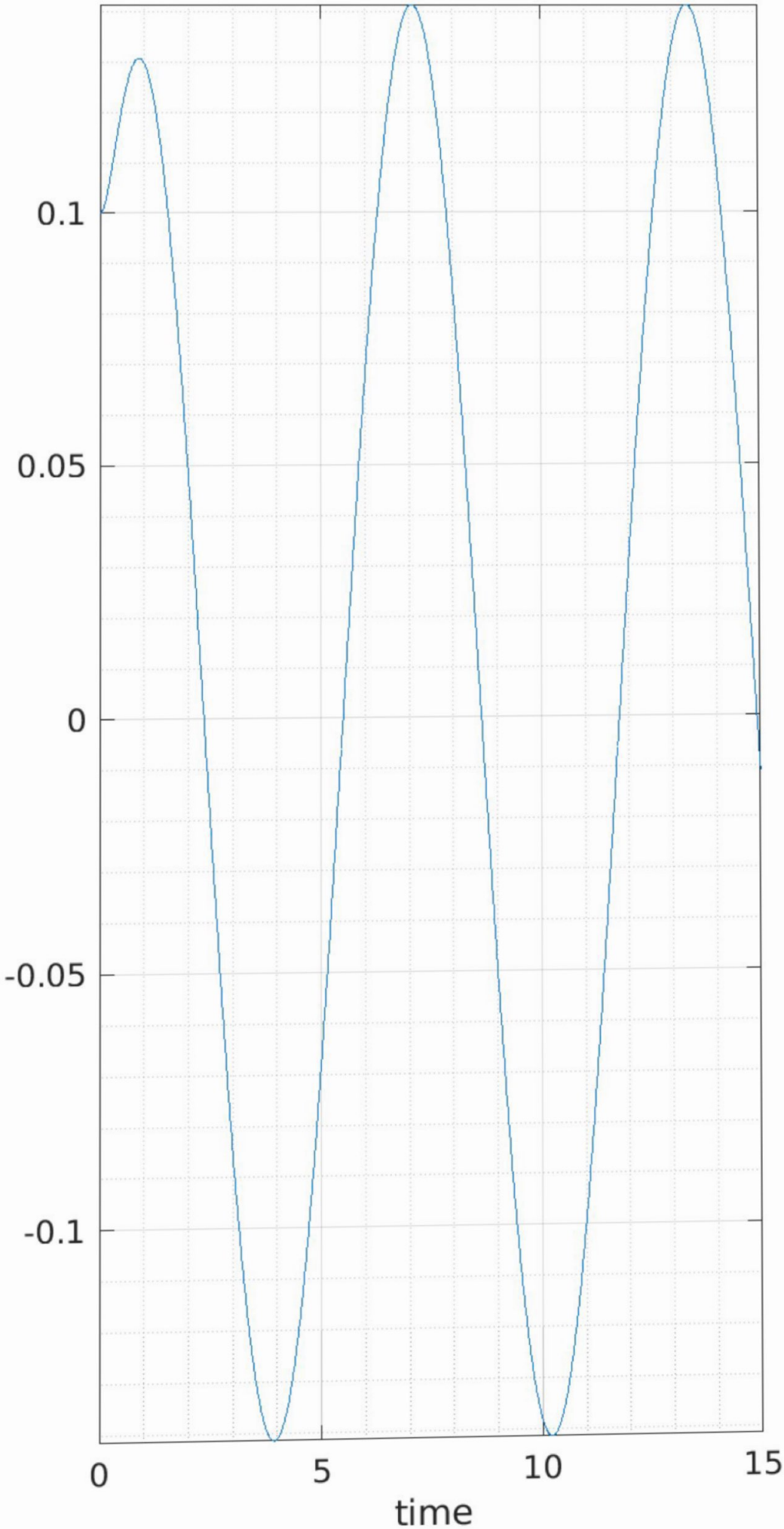
and $B = -A$

$$B = -0.1$$

General Solution = $\underline{0.1e^{-3t} - 0.1e^{-2t} + 0.1\sin t + 0.1\cos t}$

2 command window

```
clear
clc
close all
syms x t
x = 0.1*exp(-3*t) - 0.1*exp(-2*t) + 0.1*cos(t) + 0.1*sin(t)
t = 0:0.01:15
xn= subs(x)
plot(t,xn)
xlabel('time')
grid on
grid minor
axis tight
```



c $0.1\cos t + 0.1\sin t = K\sin(t+a)$ at steady flow

$0.1\cos t + 0.1\sin t = K\sin t\cos a + K\sin a\cos t$

Comparing coefficient

coefficient of $\cos t$: $0.1 = K\sin a$

coefficient of $\sin t$: $0.1 = K\cos a$

Square $K\sin a$ and $K\cos a$ and equate it to the addition

$K^2\sin^2 a + K^2\cos^2 a = 0.1 + 0.1$

$K^2(\sin^2 a + \cos^2 a) = 0.2$

$K^2 = 0.2$ $(\sin^2 a + \cos^2 a = 1)$

$K^2 = \frac{2}{100}$

$K = \frac{\sqrt{2}}{10}$

$\frac{K\sin a}{K\cos a} = \frac{0.1}{0.1}$

$\tan a = 1$

$a = \tan^{-1}(1)$

$a = 45^\circ$ or $\frac{\pi}{4}$

$\therefore$ K steady state;

$K_{ss} = \frac{\sqrt{2}}{10}\sin\left(\frac{\pi}{4} + t\right)$