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18/ENG08/003.

a. Show that the limit of the function given in Equation (1.1) as x approaches 0 is a/b.

$$f(x) = \frac{\sin a(x)}{bx} \qquad (1.1)$$

Solution:

Direct substitution:

$$\lim_{x \to 0} \frac{\sin ax}{bx} = \frac{\sin a(0)}{b(0)}$$

$$= \frac{0}{0} = 0 = \text{Indeterminant}$$

Using L'hopital's rule:

$$\lim_{x \to 0} \frac{\sin ax}{bx} = \frac{a\cos ax}{b}$$

$$= \frac{a\cos a(0)}{b} = \frac{a\cos 0}{b} = \frac{a}{b}.$$

b. The model of a system has been developed to be given in Equation (1.2).

$$f(x) = 5x - 21 \qquad (1.2)$$

Given that $\delta = 0.1$ and $\Delta\delta = 0.01$, demonstrate in tabular form, that the limit of the model as $x \to 6$ is equal to 9.

Solution:

$\delta = 0.1$, $\Delta\delta = 0.01$ $\quad 6 - 0.1 = 5.9$

$\varepsilon = 0.5$, $\Delta\varepsilon = 0.05$ $\quad 9 - 0.5 = 8.5$

$6 + 0.1 = 6.1$ $\quad 9 + 0.5 = 9.5$

$0.01 + x$	$0.05 + y$
5.9	8.5
5.91	8.55
5.92	8.6
5.93	8.65
5.94	8.7
5.95	8.75
5.96	8.8
5.97	8.85
5.98	8.9
5.99	8.95
6	9

$x - 0.01$	$y - 0.05$
6.1	9.5
6.09	9.45
6.08	9.4
6.07	9.35
6.06	9.3
6.05	9.25
6.04	9.2
6.03	9.15
6.02	9.1
6.01	9.05
6	9

c. Show whether the function given in Equation (1.3) is continuous on the interval $|-5;5|$.

$$f(x) = (25 - x^2)^{1/2} \qquad (1.3)$$

Solution:

For −5:

$\lim_{x \to -5} (25 - x^2)^{1/2} = (25 - [-5]^2)^{1/2}$
$= (25 - 25)^{1/2} = 0$

For −4:

$\lim_{x \to -4} (25 - x^2)^{1/2} = (25 - [-4]^2)^{1/2}$
$= (25 - 16)^{1/2} = 3$

For −3:

$\lim_{x \to -3} (25 - x^2)^{1/2} = (25 - [-3]^2)^{1/2}$
$= (25 - 9)^{1/2} = 4$

For −2:

$\lim_{x \to -2} (25 - x^2)^{1/2} = (25 - [-2]^2)^{1/2}$
$= (25 - 4)^{1/2} = 4.58$

For −1:

$\lim_{x \to -1} (25 - x^2)^{1/2} = (25 - [-1]^2)^{1/2}$
$= (25 - 1)^{1/2} = 4.89$

For 0:

$\lim_{x \to 0} (25 - x^2)^{1/2} = (25 - [0]^2)^{1/2} = 5$

For 1:

$\lim_{x \to 1} (25 - x^2)^{1/2} = (25 - (1)^2)^{1/2} = 4.89$

For 2:

$\lim_{x \to 2} (25 - x^2)^{1/2} = (25 - (2)^2)^{1/2} = 4.58$

For 3:

$\lim_{x \to 3} (25 - x^2)^{1/2} = (25 - (3)^2)^{1/2} = 4$

For 4:

$\lim_{x \to 4} (25 - x^2)^{1/2} = (25 - (4)^2)^{1/2} = 3$

For 5:

$\lim_{x \to 5} (25 - x^2)^{1/2} = (25 - (5)^2)^{1/2} = 0$

$\therefore$ the function $f(x) = (25 - x^2)^{1/2}$ is continuous on the interval $|-5, 5|$.