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Mechanical Engineering
ENGL381

800 LVL

$$j) \frac{d^2x}{dt^2} + 5 \frac{dx}{dt} + 6x = \cos t$$

$$M^2 + 5M + 6 = 0$$

$$M^2 + 3M + 2M + 6 = 0$$

$$M(M+3) + 2(M+3) = 0$$

$$M+2 = 0$$

$$M+3 = 0$$

$$M = -2$$

$$\text{or } M = -3$$

$$\therefore \text{C.F.} = Ae^{-2t} + Be^{-3t}$$

$$P.I. = \cos t$$

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

$$= (-C \cos t - D \sin t) + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$
$$= -C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$\cos t (-C + 5D + 6)$$

$$(-C \cos t + 5D \cos t + 6 \cos t) + (-D \sin t - 5C \sin t + 6D \sin t) = \cos t$$

$$5D \cos t + 5D \cos t = \cos t$$

$$5D \sin t - 5C \sin t = 0$$

$$5C + 5D = 1$$

$$-5C + 5D = 0$$

$$10D = 1$$

$$D = \frac{1}{10}$$

$$5C + 5D = 1$$

$$5C + \frac{5}{10} = 1$$

$$C = \frac{1}{10}$$

$$\therefore P.I = \frac{1}{10} (\cos t + \sin t)$$

$$\therefore x = C.F + P.I.$$

$$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10} (\cos t + \sin t)$$

when $t=0$ $x=0.1$ & $\frac{dx}{dt} = 0$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10} (\cos 0 + \sin 0)$$

$$0.1 = A + B - 0.1$$

$$A + B = 0.1 + 0.1$$

$$A + B = 0.2 \quad \text{--- eqn (1)}$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + \frac{1}{10} (\cos t + \sin t)$$

$$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} + \frac{1}{10} (1)$$

$$= -2A - 3B + 0.1$$

$$= 3A + 2B = 0.1 \quad \text{--- (2)}$$

$$A + B = 0.2$$

$$A = 0.2 - B \quad \text{--- (3)}$$

Sub eqn 3 into eqn (2)

$$3(0.2 - B) + 2B = 0.1$$

$$0.6 - 3B + 2B = 0.1$$

$$-B = -0.5$$

$$B = 0.5$$

$$A = -0.3$$

$$\therefore x = -0.3e^{-2t} + 0.5e^{-3t} + \frac{1}{10} (\cos t + \sin t)$$

b Command Window

clear

clc

close all

sym x; t;

t = [0 : 0.1 : 15];

x = 0.1 * (sin(t + pi/4));

plot(t, x)

$$C \quad x = k \sin(t + a).$$

$$x = 0.1$$

$$t = 0$$

$$\left\{ \frac{dx}{dt} = 0 \right.$$

$$\frac{dx}{dt} = k \cos(t + a).$$

$$0 = k \cos(0 + a).$$

$$0 = k \cos a. \quad \text{—————} \quad (1)$$

$$\cancel{\cos a = k}$$

$$0.1 = k \sin(a). \quad \text{—————} \quad (2)$$

$$k \sin a = 0.1$$

$$\cos a = 0$$

$$a = \cos^{-1}(0)$$

$$a = 90^\circ$$

Sub into eqn 2

$$0.1 = k \sin 90^\circ$$

$$k = 0.1 / \sin 90$$

$$k = 0.1$$

$$x = 0.1 \sin(t + 90^\circ).$$