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TRANSFER

$$.) \quad \frac{d^2x}{dt^2} + 5 \frac{dx}{dt} + 6x = \cos t$$

$$m^2 + 5m + 6 = 0$$

$$m^2 + 3m + 2m + 6 = 0$$

$$m(m+3) + 2(m+3) = 0$$

$$m+2 = 0 \quad m+3 = 0$$

$$m = -2 \quad \text{or} \quad m = -3$$

$$\therefore \text{C.F.} = Ae^{-2t} + Be^{-3t}$$

$$P.I. = \cos t$$

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

$$= (-C \cos t - D \sin t) + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$
$$= -C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$\cos t (-C + 5D + 6) + \sin t (-D - 5C + 6D) = \cos t$$

$$(-C \cos t + 5D \cos t + 6 \cos t) + (-D \sin t - 5C \sin t + 6D \sin t) = \cos t$$
$$5C \cos t + 5D \cos t = \cos t$$

$$5D \sin t - 5C \sin t = 0$$

$$5C + 5D = 1$$

$$-5C + 5D = 0$$

$$10D = 1$$

$$D = \frac{1}{10}$$

$$5C + 5D = 1$$

$$5C + \frac{5}{10} = 1$$

$$C = \frac{1}{10}$$

$$\therefore P \cdot I = \frac{1}{10} (\cos t + \sin t).$$

$$\therefore x = C \cdot F + P \cdot I.$$

$$x = A e^{-2t} + B e^{-3t} + \frac{1}{10} (\cos t + \sin t).$$

when $t=0$ $x = 0.1$ & $\frac{dx}{dt} = 0$

$$0.1 = A e^{-2(0)} + B e^{-3(0)} + \frac{1}{10} (\cos 0 + \sin 0)$$

$$0.1 = A + B - 0.1.$$

$$A + B = 0.1 + 0.1$$

$$A + B = 0.2 \quad \text{--- eqn (1)}$$

$$\frac{dx}{dt} = -C \sin t + D \cos t.$$

$$\frac{dx}{dt} = -2A e^{-2t} - 3B e^{-3t} + \frac{1}{10} (\cos t + \sin t).$$

$$0 = -2A e^{-2(0)} - 3B e^{-3(0)} + \frac{1}{10} (1)$$

$$= -2A - 3B + 0.1$$

$$= 3A + 2B = 0.1 \quad \text{--- (2)}$$

$$A + B = 0.2 \quad \text{--- (1)}$$

$$A = 0.2 - B \quad \text{--- (3)}$$

Sub eqn 3 into eqn (2).

$$= 3(0.2 - B) + 2B = 0.1$$

$$0.6 - 3B + 2B = 0.1$$

$$-B = -0.5$$

$$B = 0.5$$

$$A = -0.3$$

$$\therefore x = -0.3 e^{-2t} + 0.5 e^{-3t} + \frac{1}{10} (\cos t + \sin t).$$

b Command Window

clear

clc

close all

Sym x; t;

t = [0:0.1:15];

x = 0.1 * (sin(t + 90));

plot(t, x)

c $x = k \sin(t + a)$.

$$x = 0.1 \quad t = 0 \quad \left\{ \frac{dx}{dt} = 0 \right.$$

$$\frac{dx}{dt} = k \cos(t + a)$$

$$0 = k \cos(0 + a)$$

$$0 = k \cos a \quad \text{--- (1)}$$

$$\cos a = 0$$

$$0.1 = k \sin(a) \quad \text{--- (2)}$$

$$k \sin a = 0.1$$

$$\cos a = 0 \quad a = \cos^{-1}(0)$$

$$a = 90^\circ$$

Sub into eqn 2

$$0.1 = k \sin 90^\circ$$

$$k = 0.1 / \sin 90$$

$$k = 0.1$$

$$x = 0.1 \sin(t + 90^\circ)$$