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17/ENG06/045

MECHANICAL ENGINEERING

ENG 381

$$1) \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$m^2 + 5m + 6 = 0$$

$$m^2 + 3m + 2m + 6 = 0$$

$$m(m+3) + 2(m+3) = 0$$

$$m+2 = 0$$

$$m+3 = 0$$

$$m = -2$$

$$\text{or } m = -3$$

$$\therefore \text{C.F.} = Ae^{-2t} + Be^{-3t}$$

$$\text{P.I.} = \cos t$$

$$x = C\cos t + D\sin t$$

$$\frac{dx}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$$

$$= (-C\cos t - D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$$

$$= -C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$$

$$(-C\cos t + 5D\cos t + 6C\cos t) + (-D\sin t - 5C\sin t + 6D\sin t) = \cos t$$

$$5C\cos t + 5D\cos t = \cos t$$

$$5D\sin t - 5C\sin t = 0$$

$$5C + 5D = 1$$

$$-5C + 5D = 0$$

$$10D = 1$$

$$D = \frac{1}{10}$$

$$5C + 5D = 1$$

$$5C + \frac{1}{2} = 1$$

$$5C = \frac{1}{2}$$

$$C = \frac{1}{10}$$



$$\therefore P.I = \frac{1}{10} (\cos t + \sin t)$$

$$\therefore x = C.F + P.I$$

$$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10} (\cos t + \sin t)$$

$$\text{When } t = 0 \quad x = 0.1 \quad \& \quad \frac{dx}{dt} = 0$$

~~$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10} (\cos 0 + \sin 0)$$~~

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10} (\cos 0 + \sin 0)$$

$$0.1 = A + B - 0.1$$

$$A + B = 0.1 + 0.1$$

$$A + B = 0.2 \quad \text{--- eqn}$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + \frac{1}{10} (\cos t + \sin t)$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + \frac{1}{10} (\cos t + \sin t)$$

$$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} + \frac{1}{10} (1)$$

$$= -2A - 3B + 0.1$$

$$= 3A + 2B = 0.1 \quad \text{--- (2)}$$

$$A + B = 0.2$$

$$A = 0.2 - B \quad \text{--- (3)}$$

Sub eqn (3) into eqn (2)

$$= 3(0.2 - B) + 2B = 0.1$$

$$0.6 - 3B + 2B = 0.1$$

$$-B = -0.5$$

$$B = 0.5$$

$$A = -0.3$$

$$\therefore x = -0.3e^{-2t} + 0.5e^{-3t} + \frac{1}{10} (\cos t + \sin t)$$



Command Window

clear

clc

close all

Sym x; t

t = [0:0.1:15];

x = 0.1 \* (sin(t + 90))

Plot(t, x)

$$x = k \sin(t + a)$$

$$x = 0.1 \quad t = 0$$

$$\frac{dx}{dt} = k \cos(t + a)$$

dt

$$0 = k \cos(0 + a)$$

$$0 = k \cos a \quad \text{--- (1)}$$

$$0.1 = k \sin a \quad \text{--- (2)}$$

$$k \sin a = 0.1$$

$$\cos a = 0$$

$$a = \cos^{-1}(0)$$

$$a = 90^\circ$$

sub. into eqn (2)

$$0.1 = k \sin 90^\circ$$

$$k = 0.1 / \sin 90^\circ$$

$$k = 0.1$$

$$x = 0.1 \sin(t + 90^\circ)$$