KAKIRA IMRAN NURA
I/ENG02/1040
COMPUTER ENGR.

Solution

a) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} = 6x - \cos t$

Auxillary Equation : $m^2 + 6 = 0$

$m^2 + 2m + 3m + 6 = 0$

$m(m+2) + 3(m+2) = 0$

$(m+3)(m+2) = 0$

$m = -3$ or -2

C.F $x = Ae^{-3x} + Be^{-2x}$

P.I $x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = C\cos t - D\sin t$

Putting the value of $\frac{d^2x}{dt^2}$ & $\frac{dx}{dt}$ in the equation.

$-[C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t)] = \cos t$

Comparing Coefficient

Cost : $C + 5D + 6C = 1$ ——(1)

$D - 5C + 6D = 0$ ——(11)

$5C + 5D = 1$ ——(3)

$5D - 5C = 0$ ——(4)

$5D = 1 - 5C$ ——(5)

putting eqn(5) in eqn(4)

$1 - 5C - 5C = 0$

$1 - 10C = 0$

$C = 1/10 ; 0.1$

$\therefore 5D = 1 - 5C$

$5D = 0.5$

$D = 0.5/5 = 0.1$

P.S $x = Ae^{-3t} + Be^{-2t} + 0.1\sin t + 0.1\cos t$

when $t = 0$,

$$x = 0.1\frac{dx}{dt} = 0$$

$$0.1 = A + B + 0.1$$

$$A + B = 0 \quad \text{---(6)}$$

$$\frac{dy}{dx} = -3Ae^{-3t} - 2Be^{-2t} + 0.1\sin t + 0.1\cos t$$

$$0 = -3A - 2B + 0.1$$

$$B = -A$$

$$0 = -3A - 2(-A) + 0.1$$

$$0 = -A + 0.1$$

$$B = -A$$

$$B = 0.1$$

G.S $0.1A^{-3t} - 0.1e^{-2t} + 0.1\sin t + 0.1\cos t$

2) Command Window

```
Clear
clc
close all
Syms xt
x = 0.1 * exp(-3*t) - 0.1 * exp(-2*t) + 0.1 * cos(t) + 0.1sin(t)
        t = 0; 0.1 : 15
x_n = Subs(x)
Plot (t, x_n)
xlabel ('time')
grid on
grid minor
axis tight
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(c) $0.1\cos t + 0.1\sin t = K\sin(1+9)$ at steady flow

$0.1\cos t + 0.1\sin t = K\sin\cos$ at $K\sin\cos t$

Comparing Coefficient

Coefficient of $\cos t = 0.1 = K\sin\alpha$

Coefficient of $\sin t = 0.1 = K\cos\alpha$ and equate it to the addition

Square $K\sin\alpha$ and $K\cos\alpha$

$K^2\sin^2\alpha t + K^2\cos^2\alpha = 0.1 + 0.1$

$K^2(\sin^2\alpha t \cos^2\alpha) = 0.2$

$K^2 = 0.2$

$K^2 = \frac{2}{10}$ $(\sin^2\alpha + \cos^2\alpha = 1)$

$K = \frac{\sqrt{2}}{10}$

$K\sin\alpha = \frac{0.1}{0.1}$

$\tan\alpha = 1$

$\alpha = \tan^{-1}(1)$

$\alpha = 45^\circ$ or $\frac{\pi}{4}$

$\therefore$ K steady state

$K_{ss} = \frac{\sqrt{2}}{10}\sin\left(\frac{\pi + t}{4}\right)$ //