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CHEMICAL ENGINEERING

17/ENG01/013

ENGINEERING MATHEMATICS III

ASSIGNMENT I

A)

The equation of motion of a body performing damped free vibration is given as: $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$, given that when $t=0$, $x=0.1$ and $\frac{dx}{dt}=0$, obtain the soln of the eqn.

- * Write the steady-state solution in the form of $k \sin(x + a)$.
- * with the aid of MATLAB plot the relationship between x & t for using a step size of 0.01 unit.

Soln

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t \quad \text{--- (i)}$$

CF: $m^2 + 5m + 6 = 0$
 $m_1 = -2, m_2 = -3$
 $x = Ae^{m_1 t} + Be^{m_2 t}$

I

$$x = Ae^{-2t} + Be^{-3t}$$

P.I : $f(t) = \cos t$ then $x = C \cos t + D \sin t$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

$$-C \cos t - D \sin t + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$5C \cos t + 5D \sin t - 5C \sin t + 5D \cos t = \cos t$$

$$(5C + 5D) \cos t + (5D - 5C) \sin t = \cos t$$

$$5C + 5D = 1 \quad \text{--- (2)}$$

$$5D - 5C = 0 \quad \text{--- (3)}$$

from eqn (3)

$$5D = 5C$$

$$D = C$$

Subst in eqn (2)

$$5D + 5D = 1$$

$$10D = 1$$

$$D = 1/10$$

$$C = 1/10$$

P.I : $x = \frac{1}{10} \cos t + \frac{1}{10} \sin t$

General Solution : CF + PI

$$\Rightarrow x = Ae^{-2t} + Be^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

To obtain Particular Solution.

When $t=0$, $x = 0.1$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10} \cos(0) + \frac{1}{10} \sin(0)$$

$$0.1 = A + B + \frac{1}{10}$$

$$A + B = 0 \quad \text{--- (4)}$$

To get $\frac{dx}{dt}$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + \frac{1}{10} \sin t + \frac{1}{10} \cos t$$

when $t=0$, $\frac{dx}{dt} = 0$

$$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} + \frac{1}{10} \sin(0) + \frac{1}{10} \cos(0)$$

$$0 = -2A - 3B + \frac{1}{10}$$

$$2A + 3B = \frac{1}{10} \quad \text{--- (5)}$$

from eqn (4)

$$A = -B$$

$$2(-B) + 3B = \frac{1}{10}$$

$$-2B + 3B = \frac{1}{10}$$

$$B = \frac{1}{10}$$

$$A = -\frac{1}{10}$$

$$P.S : x = \frac{-1e^{-2t}}{10} + \frac{1e^{-3t}}{10} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$x = \frac{1}{10} (e^{-3t} - e^{-2t} + \cos t + \sin t)$$

B) MATLAB mfile

Commandwindow

Clear

clc

close all

syms t

t = 0:0.001:15

x = 0.1 * (exp(-3 * t) - exp(-2 * t) + cos(t) + sin(t))

$x_n = \text{subs}(x)$

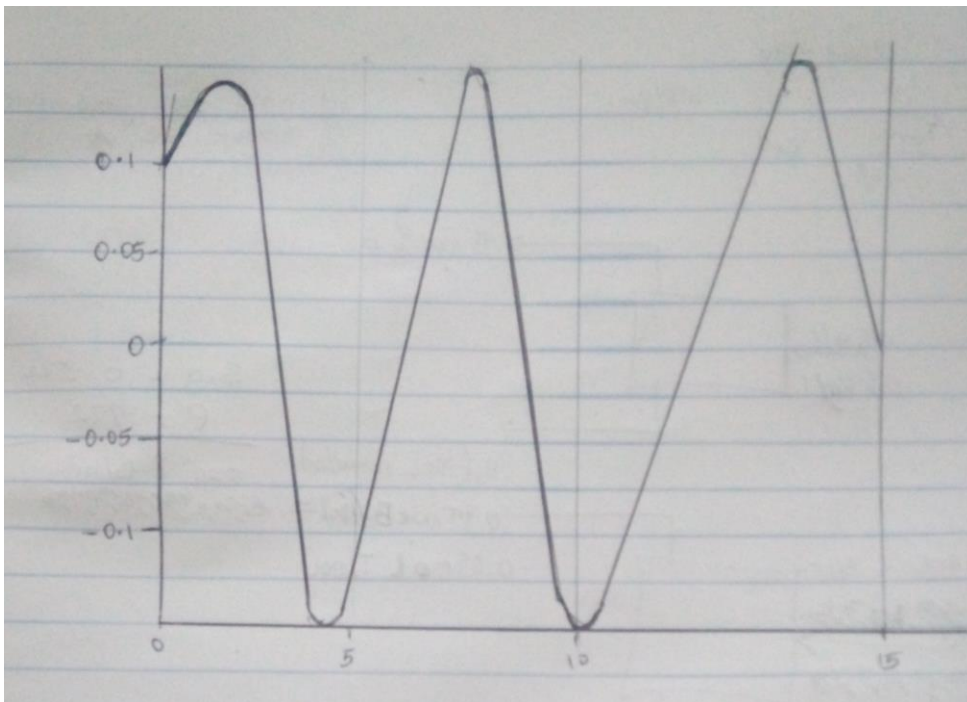
$\text{plot}(t, x_n)$

axis tight

grid on

grid minor

GRAPH



C) $x = k \sin(t + a)$

$X = 0.1$ at $t=0$ and $dx/dt = 0$

$dx/dt = k \cos(t+a)$

$0 = k \cos(0+a) ; k \cos(a) = 0$ -----(i)

$0.1 = k \sin(0+a) ; k \sin(a) = 0.1$ -----(ii)

$\cos(a) = 0 ; a = 90$

Sub a in eqn(i)

$0.1 = k \sin(90)$

$K = 0.1/\sin(90) = 0.1$

$x = 0.1(\sin(t+90))$

MATLAB

Commandwindow

Close all

Clear

Clc

Syms t , x

t = (0:0.001:15)

x=0.1 * (sin(t+90))

plot(t,x)