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MATRIC NO: 17/ENG05/030

DEPARTMENT: MECHATRONICS ENGINEERING

COURSE: ENG 381 (ENGINEERING MATHEMATICS III)

Assignment I

1.) The dynamic model of a body in motion performing damped force vibrations is as eqn (1)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos(t)$$

Given that when $t = 0$, $x = 0.1$ $dx/dt = 0$

a) Using the auxillary equation method, obtain the solution of the model in form of an expression having x as a function of t,

b) With the aid of a MATLAB mfile program, plot the relationship between x and t for $0 \leq t \leq 15$ time unit using a step size of 0.01 unit and

c) Write the steady state solution of the model in form of $x = 16\sin(t+a)$.

Solution

$C.F = m^2 + 5m + 6 = 0$

$(m+3)(m+2) = 0$

$m = -3$, $m = -2$

$x = Ae^{-3t} + Be^{-2t}$

$f(t) = \cos t$

$x = C \cdot \cos t + D\sin t$

$\frac{dx}{dt} = -C \cdot \sin t + D \cdot \cos t$

$\frac{d^2x}{dt^2} = -C \cdot \cos t - D\sin t$

Subs.

$= -C \cdot \cos t - D\sin t + 5(-C \cdot \sin t + D \cdot \cos t) + 6(C \cdot \cos t + D\sin t) = \cos t$

$= \cos t(-C + 5D + 6C) + \sin t(-D - 5C + 6D) = \cos t$

$= \cos t(5D + 5C) + \sin t(-5C + 5D) = \cos t$

$5D + 5C = 1$

$5D - 5C = 0$

$10D = 1$; $D = \frac{1}{10}$

to Find C

$$5\left(\frac{1}{10}\right) + 5C = 1$$

$$\frac{1}{2} + 5C = 1$$

$$C = \frac{1}{10}$$

$$G.S = x = Ae^{-3t} + Be^{-2t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$$

Subs. $(x = 0.1)$ $t = 0$

$$0.1 = Ae^{-0} + Be^{-0} + \frac{1}{10}\cos 0 + \frac{1}{10}\sin 0$$

$$0.1 = A + B + 0.1$$

$$A + B = 0 \text{ — eqn (1)}$$

$$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$$

$$0 = -3A - 2B + \frac{1}{10}$$

$$3A + 2B = \frac{1}{10} \quad \text{--- eqn (II)}$$

$$A + B = 0 \quad \text{---- } \times 2$$

$$3A + 2B = 0.1 \quad \text{--- } \times 1$$

$$2A + 2B = 0$$

$$3A + \cancel{2B} = 0.1$$

$$-A = -0.1$$

$$A = 0.1$$

To Find B

$$0.1 + B = 0$$

$$B = -\frac{1}{10} \text{ or } -0.1$$

$$G.S = -\frac{1}{10}e^{-3t} + \frac{1}{10}e^{-2t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$$

$$G.S = \frac{1}{10}e^{-2t} - \frac{1}{10}e^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$$

$$= \frac{1}{10}\left(e^{-2t} - e^{-3t} + \cos t + \sin t\right)$$

III) MATLAB m file

Commandwindow

```
clear
clc
close all
syms t
t = 0:0.01:15
x = 0.1 * [exp(-2*t) - exp(-3*t) + cos(t) + sin(t)]
xn = subs(x)
plot(t, xn)
axis tight
grid on
grid minor
```