

Arunima Sanyal BSc
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Mathematical Engineering

$$\text{Auxiliary equation} = m^2 + 5m + 6 = 0$$

$$m^2 + 3m + 2m + 6 = 0$$

$$m(m+3) + 2(m+3) = 0$$

$$(m+2)(m+3) = 0$$

$$m_1 = -2 \quad m_2 = -3$$

$$CF = Ae^{-2x} + Be^{-3x}$$

$$PI = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

Putting the value $\frac{d^2x}{dt^2} + 5 \frac{dx}{dt} + 6x$ in the eqn.

$$-C \cos t - D \sin t + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

Comparing Coeffs

$$\text{Coefficient of } \cos t: \frac{1}{1}C + 5D + 6C = 1$$

$$\text{Coefficient of } \sin t: D - 5C + 6D = 0$$

$$\text{from eqn (1)} \quad 5C + 5D = 1$$

$$\text{from eqn (2)} \quad 5D + 5C = D$$

$$\text{from eqn (1)} \quad 5D = 1 - 5C$$

$$\text{Put eqn (5) in eqn (4)}$$

$$1 - 5C - 5C = 0$$

$$1 - 10C = 0$$

$$C = 1/10$$

$$5D = 1 - 5C$$

Put the value of C in eqn (5)

$$5D = 1 - 5(0.1)$$

$$5D = 0.5$$

$$D = 0.5/5 = 0.1$$

Particular Solution : $x = Ae^{-3t} + Be^{-2t} + 0.1 \sin t + 0.1 \cos t.$

when $t=0$ $x = 0.1$ $\frac{dx}{dt} = 0.$

$$0.1 = A + B + 0.1$$

$$A + B$$

$$dy = 3Ae^{-3t} - 2Be^{-2t} + 0.1 \sin t + 0.1 + \cos t.$$

$$0 = -3A - 2B + 0.1$$

from eqn (6) $B = -A.$

$$0 = -3A - 2(-A) + 0.1$$

$$0 = -A + 0.1$$

$$A = 0.1$$

and $B = -A.$

$$B = -0.1$$

$$G.S = 0.1e^{-3t} - 0.1e^{-2t} + 0.1 \sin t + 0.1 \cos t.$$

b.) Command window

clear

clc

close all

syms t

$$x = 0.1 * \exp(-3 * t) - 0.1 * \exp(-2 * t) + 0.1 * \cos(t) + 0.1 * \sin(t)$$

$$t = 0 : 0.01 : 15$$

$$hx = \text{subs}(t, x)$$

$$\text{plot}(t, hx)$$

$$\text{xlabel}('time')$$

grid on

grid minor

axis tight

$$c) 0.1 \cos t + 0.1 \sin t = k (\sin(t + \alpha))$$

$$0.1 \cos t + 0.1 \sin t = k \sin t \cos \alpha + k \cos t \sin \alpha$$

Comparing Coefficient

$$\text{Coefficient of } \cos t = k \cos \alpha = 0.1$$

$$\text{Coefficient of } \sin t = k \sin \alpha = 0.1$$

Square both $k \cos \alpha$ and $k \sin \alpha$ and equate to the addition:

$$k^2 \cos^2 \alpha + k^2 \sin^2 \alpha = 0.1^2 + 0.1^2$$

$$k^2 (\cos^2 \alpha + \sin^2 \alpha) = 0.2$$

$$k^2 = 0.2$$

$$k = \frac{2}{100}$$

$$k = \frac{\sqrt{2}}{10}$$

$$\frac{k \sin \alpha}{k \cos \alpha} = \frac{0.1}{0.1}$$

$$\tan \alpha = 1$$

$$\tan \alpha = 1$$

$$\alpha = \tan^{-1}(1)$$

$$\alpha = 45^\circ \text{ or } \frac{\pi}{4}$$

k steady state;

$$k_{ss} = \frac{\sqrt{2}}{10} \sin\left(\frac{\pi}{4} + t\right)$$