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DEPARTMENT - MECHANICAL
LEVEL - 300.
COURSE - ENG 381

The dynamic model of a body in motion performing forces vibration is a sin equation.

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that $t=0$ $x=0.1$ and $dx/dt = 0$.

a) Using the auxilary equation method, obtain the solution of the model information of an expression having x as a function of t.

b) With the aid of MATLAB mfile program, plot the relationship between x and t for $0 \leq t \leq 15$ time int using a step size of 0.01 unit end.

c) Write the steady state solution of the model inform of $x = k\sin(t-a)$

## Solution

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t.$$

In auxillary form.

$$m^2 + 5m + 6 = 0.$$

$$(m^2 + 2m) + (3m + 6) = 0$$

$$m(m+2)\ 3(m+2) = 0.$$

$$m = -2 \quad m = -3.$$

$$x = Ae^{-2t} + Be^{-3t}.$$

$$P.I = \cos t.$$

$$x = C\cos t + D\sin t.$$

$$dx/dt = -C\sin t + D\cos t.$$

$$d^2x/dt^2 = -C\cos t - D\sin t$$

Substituting back into equation

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$-C\cos t + 6C\cos t - D\sin t + 6D\sin t - 5C\sin t + 5D\cos t = \cos t$

$5C\cos t + 5D\cos t + 5D\sin t - 5C\sin t + 5D\cos t = \cos t$

$5C\cos t + 5D\cos t + 5D\sin t - 5C\sin t = \cos t$

$5C\cos t + 5D\cos t = \cos t.$

$5C + 5D = 1$ (1)

$5D\sin t - 5C = 0$.

$5D - 5C = 0$ (2).

Using simultaneous equation

$5C + 5D = 1$

$-5C + 5D = 0$.

$10D = 1$

$D = 1/10$.

$5C + 5(1/10) = 1$

$5C + 1/2 = 1$

$5C = 1 - 1/2$.

$5C = 1/2$

$C = 1/10$.

$= Ae^{-2t} + Be^{-3t} + 1/10[\sin t + \cos t]$

When $t = 0$ $x = 0.1$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + 1/10[\sin(0) + \cos(0)$

$0.1 = A + B + 0 + 1/10$

$A + B = 0$ (1)

when $t = 0$ $dx/dt = 0$.

$dx/dt = -2Ae^{-2t} - 3Be^{-3t} + 0.1(\cos t - \sin t)$

$0 = -2A - 3B + 0.1$

$-0.1 = -2A - 3B$.

Remember $A + B = 0$ (iii)

$A = -B$ (iv)

$-0.1 = -2(-B) - 3B$

$-0.1 = 2B - 3B$.

$-0.1 = -B$.

$B = 0.1$

Knowing that $A = -B$.

$A = -0.1$

$x = -0.1e^{-2t} + 0.1e^{-3t} + 1/10[\sin t + \cos t]$ 0.

$x = -\frac{1}{10}e^{-2t} + \frac{1}{10}e^{-3t} + \frac{1}{10}(\sin t + \cos t)$

2 Command window.

clear.

clc

```
close all.
Syms t.
x=(1/10*exp(-2*t)-(1/10*exp(-3*t)+(1/10(Sin t)+Cos t(*)
t=0:0.01:15.
xt=Subs(x,t)
xtn=double(xt)
plot(t,x,tn)
xlabel('t')
ylabel('x')
grid on
grid minor.
grid right.
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a) Steady state

$x \to 0$ = steady state $= 0.1\cos t + 0.1\sin t$

$0.1\cos t + 0.1\sin t = K\sin(t+a)$

$K\sin(t+a) = K\sin t\cos a + K\cos t\sin a$

NB → coefficient of $\cos t = K\sin a$.

" " $= K\cos a$.

When squaring both sides.

$K^2 \sin^2 a + K^2 \cos^2 a = 0.1^2 + 0.1^2$.

$K^2(\sin^2 a + \cos^2 a) = 0.02$

$K^2 = 0.02$

$K = \sqrt{0.02}$

$K = 0.144 = \sqrt{2}/10$

$$\frac{K \sin a}{K \cos a} = \frac{0.1}{0.1} = 1$$

Remember that $\sin / \cos = \tan$.

$\tan a = 1$

$\tan^{-1}(1) = a$.

$a = 45^0$ or $\pi/4$ radian

Steady state

$= \sqrt{2}/10 \sin(t + \pi/\omega)$