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BIOMEDICAL ENGINEERING.

17/EN408/004

EN4381 ASSIGNMENT

9. $\frac{d^2x}{dt^2} + 5 \frac{dx}{dt} + 6x = \cos t$

$$x = C_f + P.I.$$

$$x = C_f.$$

$$\frac{d^2x}{dt^2} + 5 \frac{dx}{dt} + 6x = 0.$$

$$m^2 + 5m + 6 = 0$$

$\underbrace{\hspace{10em}}_{6m^2} \quad 2m, 3m.$

$$m^2 + 2m + 3m + 6 = 0$$

$$m(m+2) + 3(m+2) = 0$$

$$(m+2)(m+3) = 0$$

$$m_2 = -2, m_3 = -3$$

$$x = Ae^{-2t} + Be^{-3t}.$$

$$x = P.I$$

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

Substitute in General equation.

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$\cos t (-C + 5D + 6C) + \sin t (-D - 5C + 6D) = \cos t$$

$$\cos t (5C + 5D) + \sin t (5D - 5C) = \cos t$$

Collect the like coefficients

$$\cos t (5C + 5D) = \cos t \quad \text{--- (i)}$$

$$\sin t (5D - 5C) = 0 \quad \text{--- (ii)}$$

Divide both equations by $\cos t$ & $\sin t$

respectively.

$$5C + 5D = 1 \quad \text{--- (i)}$$

$$5D - 5C = 0 \quad \text{--- (ii)}$$

re-write eqn (ii)

$$5D = 5C$$

$$D = C$$

Put $D = C$ in eqn (i)

$$5C + 5C = 1$$

$$10C = 1$$

$$C = 1/10$$

$$\therefore D = 1/10$$

$$x = 1/10 \cos t + 1/10 \sin t$$

$$x = 1/10 (\cos t + \sin t)$$

$$x = A e^{-3t} + B e^{2t} + 1/10 (\cos t + \sin t)$$

$$\text{at } t=0; x=0.1$$

$$0.1 = A e^{-3(0)} + B e^{-2(0)} + 1/10 (\cos 0 + \sin 0)$$

$$0.1 = A + B + 1/10$$

$$-A+B = 0.1 - 1/10$$

$$-A+B = 0 \quad \text{--- (i)}$$

When $t=0$ $dx/dt = 0$; differentiating x we have
$$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} - \frac{1}{20}\sin t + \frac{1}{10}\cos t$$

$$0 = -3Ae^{-3(0)} - 2Be^{-2(0)} - \frac{1}{10}\sin 0 + \frac{1}{10}\cos 0$$

$$0 = -3A - 2B + \frac{1}{10}$$

$$3A + 2B = \frac{1}{10} \quad \text{--- (ii)}$$

∴ write eqn (i).

$$-A = -B$$

∴ put $A = -B$ in eqn (ii)

$$B(-B) + 2B = \frac{1}{10}$$

$$-3B + 2B = \frac{1}{10}$$

$$-B = \frac{1}{10}$$

$$B = -\frac{1}{10}$$

$$A = \frac{1}{10}$$

$$\therefore x = \frac{1}{10}e^{-3t} - \frac{1}{10}e^{-2t} + \frac{1}{10}(\cos t + \sin t)$$

$$x = \frac{1}{10}(e^{-3t} - e^{-2t} + \cos t + \sin t)$$

b. Command window.

clear

clc.

close all

syms t

$$x = 1/10 * (\exp(-3*t) - \exp(2*t) + \cos(t) + \sin(t))$$

$$t = 0:0.01:15$$

$$xt = subs(x, t)$$

$$xtn = double(xt)$$

plot(t, xtn)

xlabel('t')

ylabel('x')

grid on

grid minor

axis tight

