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Assignment.

1) The dynamic model of a body in motion performing damped force vibration is an equation (1)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

a) Given that when $t = 0$, $x = 0.1$ and $\frac{dx}{dt} = 0$.

b) Using the auxiliary equation obtained the solution of the model in form of an expression having x as a function of t.

b) With aid of a MATLAB mfile program plot the relationship between x and t for $0 \leq t \leq 15$ time unit using a step size of 0.01 unit

c) Write the steady state solution of the model in form of $x = k\sin(t + a)$

Solution

a) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

Auxiliary Equation: $m^2 + 5m + 6 = 0$

$$m^2 + 2m + 3m + 6 = 0$$
$$m(m+2) + 3(m+2) = 0$$
$$(m+3)(m+2) = 0$$
$$m = -3 \text{ or } -2$$

Complementary function $x = Ae^{-3x} + Be^{-2x}$

Particular integral: Assume $x = C\cos t + D\sin t$.

$$\frac{dx}{dt} = -C\sin t + D\cos t.$$

$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t.$$

Putting the value of d^2x/dt^2 & dx/dt in the equation

$-\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

Comparing Coefficient

Coefficient of cost :- $C + 5D + 6C = 1$ ..... (1)

Coefficient of sint :- $-D - 5C + 6D = 0$ ..... (2)

from eqn (1) $5C + 5D = 1$ ..... (3)

from eqn (2) $5D - 5C = 0$ ..... (4)

from eqn (1) $5D = 1 - 5C$ ..... (5)

Put equation (5) in equation (4)

$1 - 5C - 5C = 0$

$1 - 10C = 0$

$C = 1/10 ; 0.1$

$\therefore 5D = 1 - 5C$

Put the value of C in equation (5)

$5D = 0.5$

$D = \frac{0.5}{5} = 0.1$

Particular eq. solution : $x = Ae^{-3t} + Be^{-2t} + 0.1\sin t + 0.1\cos t$

when $t = 0$; $x = 0.1$ $\frac{dx}{dt} = 0$

$0.1 = A + B + 0.1$

$A + B = 0$ ..... (6)

$dy/dx = -3Ae^{-3t} - 2Be^{-2t} + 0.1\sin t + 0.1\cos t$

$0 = -3A - 2B + 0.1$

from eqn (6) $B = -A$

$0 = -3A - 2(-A) + 0.1$

$0 = -A + 0.1$

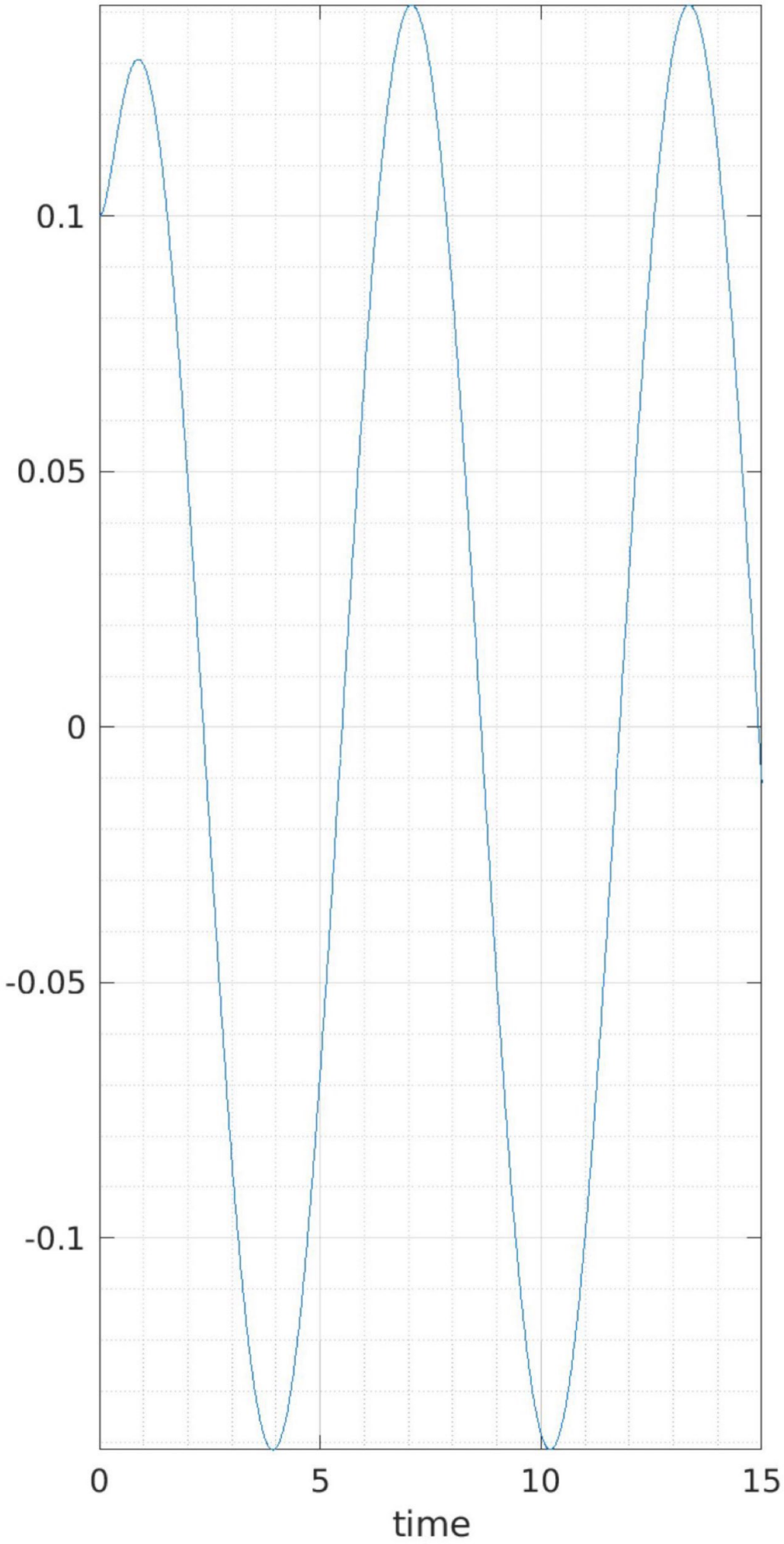
and $B = -A$

$B = 0.1$

General Solution $= 0.1e^{-3t} - 0.1e^{-2t} + 0.1\sin t + 0.1\cos t$

2) Command Window

```
clear
clc
close all
syms xct
xc = 0.1*exp(-3*t) - 0.1*exp(-2*t) + 0.1*cos(t) + 0.1*sin(t)
t = 0:0.1:15
xn = subs(xc
plot(t,xn)
xlabel('time')
grid on
grid minor
axis tight
```



c $0.1\cos t + 0.1\sin t = k\sin(1+q)$ at steady flow

$0.1\cos t + 0.1\sin t = k\sin\cos at\ k\sin\cos t$

Comparing Coefficient

Coefficient of $\cos t = 0.1 = k\sin q$

Coefficient of $\sin t = 0.1 = k\sin q$

Square $k\sin q$ and $k\cos q$ and equate it to the addition.

$$k^2\sin^2 at + k^2\cos^2 q = 0.1 + 0.1$$

$$k^2\sin^2 at\ (\cos^2 a) = 0.2$$

$$k^2 = 0.2$$

$$k^2 = \frac{2}{100} \qquad (\sin^2 q + \cos^2 q = 1)$$

$$k = \frac{\sqrt{2}}{10}$$

$$k\sin q = \frac{0.1}{0.1}$$

$$\tan q = 1$$

$$q = \tan^{-1}(1)$$

$$q = 45^\circ \text{ or } \frac{\pi}{4}$$

∴ k steady state;

$$k_{ss} = \frac{\sqrt{2}}{10}\sin\left(\frac{\pi}{4} + t\right) //$$