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Assignment

1 The dynamic model of a body in motion performing damped free vibration is an equation (i) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

(a) Using the auxillary equation method, obtain the solution of the model in form of an expression having x as a function of t.
(b) With aid of MATHLab maple program plot the relationship between x and t for $0 \leq t \leq 15$ time unit using a step size of 0.01 unit and
(c) write the steady state solution of the model in form of $x = k\sin(t+a)$

Solution:

a $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

Auxillary Equation: $m^2 + 5m + 6 = 0$

$= m^2 + 2m + 3m + 6 = 0$

$m(m+2) + 3(m+2) = 0$

$m = -3$ or -2

Complementary function: $x = Ae^{-3t} + Be^{-2t}$

Particular Integral Assume $x = C\cos t + D\sin t$

$\therefore \frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

Putting the value of $\frac{d^2x}{dt^2}$; $\frac{dx}{dt}$ in the equation

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

Comparing coefficient

Coefficient of cost: $-C + 5D + 6C = 1$ - - - - (i)

✓ ✓ sint: $-D - 5C + 6D = 0$ - - - - - (ii)

from eqn i $5C + 5D = 1$ - - - - - - - (iii)

from eqn ii $5D - 5C = 0$ - - - - - - (iv)

from eqn (i) $5D = 1 - 5C$ ------- (v)

Put eqn (v) in eqn (iv)

$1 - 5C - 5C = 0$

$1 - 10C = 0$

$C = 1/10 ; 0.1$

$\therefore 5D = 1 - 5C$

Put the value of C in eqn (v)

$5D = 1 - 5(0.1)$

$D = \frac{0.5}{5} = 0.1$

Particular solution; $x = Ae^{-3t} + Be^{-2t} + 0.1\sin t + 0.1\cos t$

when $t = 0$; $x = 0.1$, $\frac{dx}{dt} = 0$

$0.1 = A + B + 0.1$

$A + B = 0$ ---------- (vi)

$\frac{dy}{dx} = -3Ae^{-2t} - 2Be^{-2t} + 0.1\sin t + 0.1\cos t$

$0 = -3A - 2B + 0.1$

from eqn (vi) $B = -A$

$0 = -3A - 2(-A) + 0.1$

$0 = -A + 0.1$

$A = 0.1$

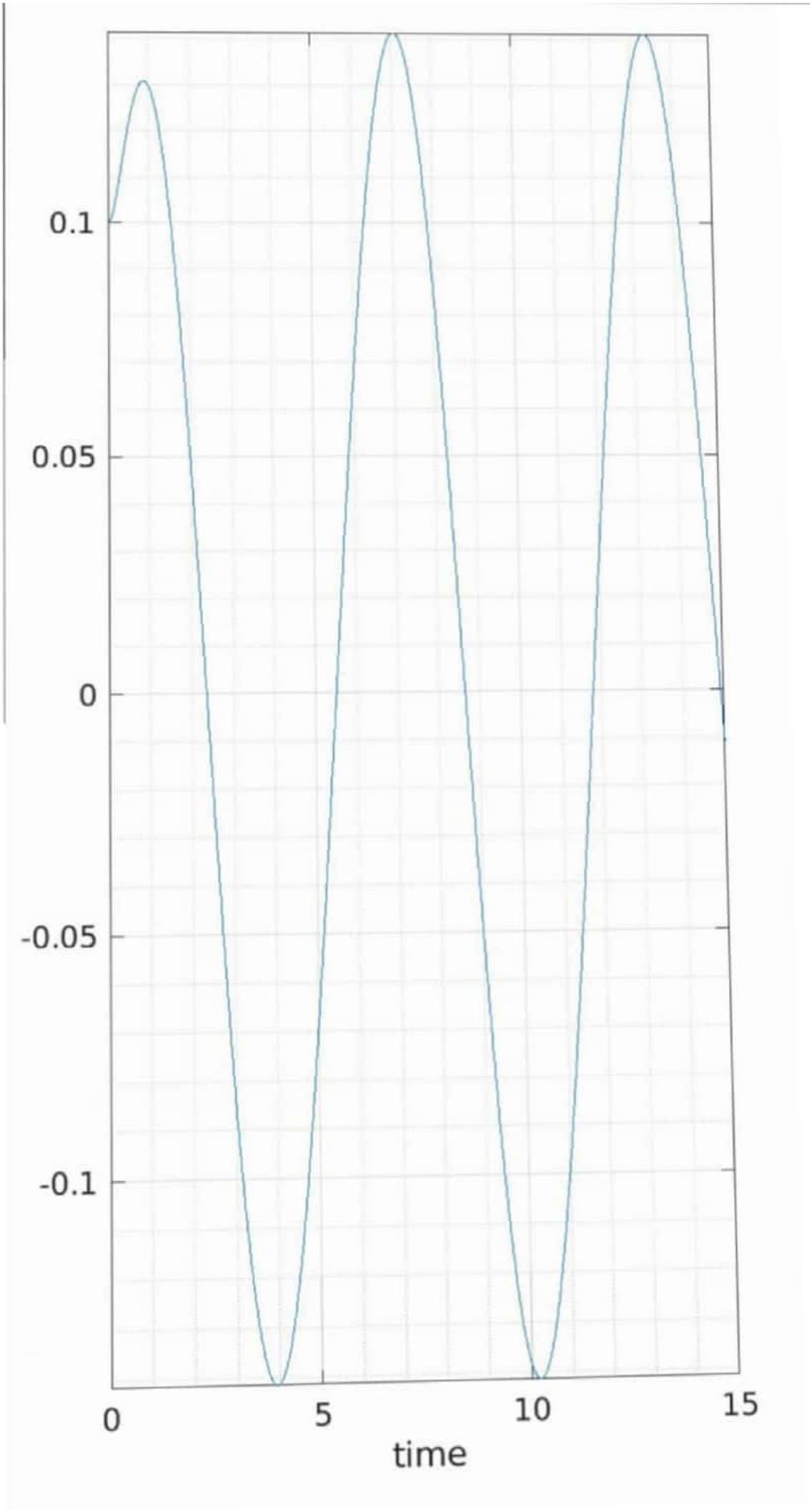
and $B = -A$

$B = -0.1$

. General Solution: $x = 0.1e^{-3t} - 0.1e^{-2t} + 0.1\sin t + 0.1\cos t$

2

command window
clear
clc
close all
syms x t
x = 0.1*exp(-3*t) - 0.1*exp(2*t) + 0.1*cos(t) + 0.1*sin(t)
t = 0:0.01:15
xn = subs(x)
plot (t,xn)
xlabel('time')
grid on
grid minor
axis tight



c $0.1\cos t + 0.1\sin t = K\overset{\sin}{\underset{\wedge}{}}(t+a)$ [steady flow]

$0.1\cos t + 0.1\sin t = K\sin t\cos a + K\sin a\cos t$

Comparing coefficient

Coefficient of $\cos t$: $0.1 = K\sin a$

Coefficient of $\sin t$: $0.1 = K\cos a$

Square $K\sin a$ and $K\cos a$ and equate it to the addition

$K^2\sin^2 a + K^2\cos^2 a = 0.1 + 0.1$

$K^2(\sin^2 a + \cos^2 a) = 0.2$

$K^2 = 0.2$ $[\sin^2 a + \cos^2 a = 1]$

$K^2 = \frac{2}{100}$

$K = \frac{\sqrt{2}}{10}$

$\frac{K\sin a}{K\cos a} = \frac{0.1}{0.1}$

$\tan a = 1$

$a = \tan^{-1}(1)$

$a = 45^\circ$ or $\frac{\pi}{4}$

$\therefore x$ steady state:

$$x_{ss} = \frac{\sqrt{2}}{10}\sin\left(\frac{\pi}{4} + t\right)$$