

OGBONNA WISDOM OKORO
17/ENG03/037
CIVIL ENG.

1) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

at homogeneity

$$m^2 + 5m + 6 = 0$$

$$(m+2)(m+3) = 0$$

$$m_1 = -2 \quad m_2 = -3$$

$$C.F = Ae^{-2t} + Be^{-3t}$$

P.I $\Rightarrow x = \cos t$

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

$$-C \cos t - D \sin t + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$5C \cos t + 5D \cos t + 5D \sin t - 5C \sin t$$

$$(5C + 5D) \cos t + (5D - 5C) \sin t = \cos t + 0 \sin t$$

$\cos t$; $5C + 5D = 1$

$\sin t$; $5D - 5C = 0 \quad \therefore 5D = 5C$

$$5C + 5C = 1$$

$$10C = 1 \quad \therefore C = 0.1$$

$$D = C = 0.1$$

$$x = 0.1 \cos t + 0.1 \sin t$$

$$x = Ae^{-2t} + Be^{-3t} + 0.1 \cos t + 0.1 \sin t$$

$$t=0, x=0.1, \frac{dx}{dt} = 0$$

at $t=0$

$$0.1 = A + B + 0.1$$

$$A + B = 0 \quad \text{--- (1)} \quad \therefore A = -B$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - 0.1 \sin t + 0.1 \cos t$$

$$0 = -2A - 3B + 0.1$$

$$2A + 3B = 0.1$$

$$2(-B) + 3B = 0.1 \quad \therefore B = 0.1 \quad \& \quad A = -0.1$$

therefore, $x = -0.1e^{-2t} + 0.1e^{-3t} + 0.1\cos t + 0.1\sin t$

2) command window

clear

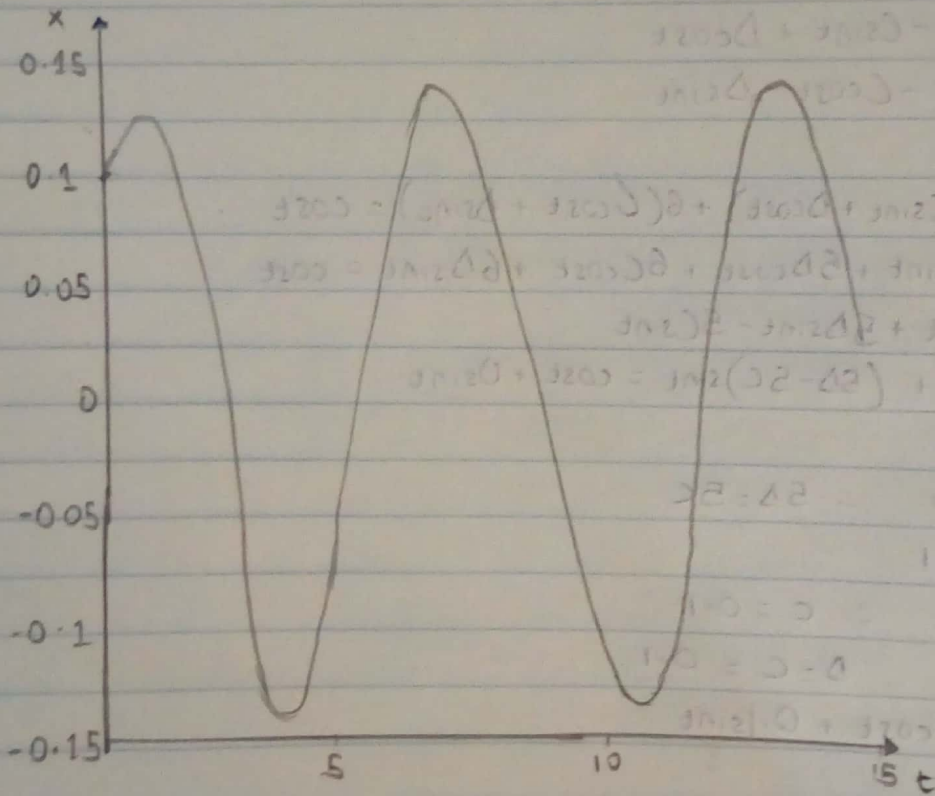
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syms t

t = 0:0.01:15

$x = -0.1 * \exp(-2*t) + 0.1 * \exp(-3*t) + 0.1 * \cos[t] + 0.1 * \sin[t]$



$$c) \quad x = K \sin(t + a)$$

at steady state;

$$0.1 \cos t + 0.1 \sin t = K \sin(t + a)$$

$$0.1 \cos t + 0.1 \sin t = K \sin t \cos a + K \cos t \sin a$$

$$\cos t: \quad 0.1 = K \sin a$$

$$\sin t: \quad 0.1 = K \cos a$$

square $K \sin a$ & $K \cos a$ and add them

$$K^2 \cos^2 a + K^2 \sin^2 a = (0.1)^2 + (0.1)^2$$

$$K^2 (\sin^2 a + \cos^2 a) = 0.02$$

from trig identities, $\sin^2 a + \cos^2 a = 1$

$$\therefore K^2 = 0.02$$

$$K = \sqrt{0.02}$$

divide $K \sin a$ by $K \cos a$

$$\frac{K \sin a}{K \cos a} = \frac{0.1}{0.1}$$

$$\tan a = 1$$

$$a = \tan^{-1}(1)$$

$$a = 45^\circ \text{ or } \frac{\pi}{4}$$

$$(2) \text{ steady state, } x = \sqrt{0.02} \sin\left(\frac{\pi}{4} + t\right)$$