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Course Name/Subject: Engineering Mathematics.

The dynamic model of a body in motion, performing damped forced vibration is as in Equation (i)

$$\frac{d^2x}{dt^2} + s \frac{dx}{dt} + Gx = c \cos t$$

Given that $t=0$, $x=0.1$ and $\frac{dx}{dt}=0$

- Using the Auxiliary Equation method, obtain the solution of the model in form of an expression having x as a function of t .
- With the use of matlab file program, plot the relationship between x and t for $0 \leq t \leq 15$ time unit using a step size of 0.01 unit and;
- Write the steady state solution of the model in form of $x = k \sin(t - a)$

Solution.

$$\frac{d^2x}{dt^2} + s \frac{dx}{dt} + Gx = c \cos t$$

Auxiliary form:

$$m^2 + sm + G = 0$$

$$(m+2)(m+3) = 0$$

$$m+2=0 \text{ or } m+3=0$$

$$m = -2 \text{ or } -3$$

$$x = Ae^{-2t} + Be^{-3t}$$

$$p \cdot I = c \cos t$$

$$x = -c \cos t + D \sin t$$

$$\frac{dx}{dt} = c \sin t + D \cos t$$

$$\frac{dx^2}{dt^2} = -c \cos t - D \sin t$$

Substituting back into the equation.

$$-c \cos t - D \sin t + s(-c \sin t + D \cos t) + G(-c \cos t + D \sin t) = c \cos t$$

$$-c \cos t - D \sin t - s c \sin t + s D \cos t + G c \cos t + G D \sin t = c \cos t$$

$$-c \cos t + G c \cos t - D \sin t + G D \sin t - s c \sin t + s D \cos t = c \cos t$$

$$s c \cos t + G D \sin t - s c \sin t + s D \cos t = c \cos t$$

$$s c \cos t + s D \cos t + s D \sin t - s c \sin t = c \cos t$$

$$5c \cos t + 5D \sin t = \cos t$$

$$5c + 5D = 1 \quad \text{--- (i)}$$

$$5D \sin t - 5c \sin t = \cos t$$

$$5D - 5c = 0 \quad \text{--- (ii)}$$

Using Simultaneous Equation.

$$5c + 5D = 1 \quad \text{--- (i)}$$

$$-5c + 5D = 0 \quad \text{--- (ii)}$$

$$10D = 1$$

$$D = \frac{1}{10}$$

$$5c + 5D = 1$$

$$D = \frac{1}{10}$$

$$5c + 5\left(\frac{1}{10}\right) = 1$$

$$5c + \frac{1}{2} = 1$$

$$5c = 1 - \frac{1}{2}$$

$$5c = \frac{1}{2}$$

$$c = \frac{1}{10}$$

$$\text{General Solution} = Ae^{-2t} + Be^{-3t} + \frac{1}{10} [\sin t + \cos t]$$

$$\text{When } t=0, x=0.1$$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10} [\sin(0) + \cos(0)]$$

$$0.1 = A + B + 0 + \frac{1}{10}$$

$$A + B = 0 \quad \text{--- (i)}$$

$$\text{When } t=0, \frac{dx}{dt} = 0$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + 0.1(\cos t - \sin t)$$

$$0 = -2A - 3B + 0.1$$

$$-0.1 = -2A - 3B \quad \text{--- (ii)}$$

Remember that $A + B = 0$ (iii)

$$A = -B \quad \text{--- (iv)}$$

$$-0.1 = -2(-B) - 3B$$

$$-0.1 = 2B - 3B$$

$$-0.1 = -B$$

$$B = 0.1$$

Knowing that $A = -B$

$$A = -0.1$$

$$\therefore x = -0.1e^{-2t} + 0.1e^{-3t} + \frac{1}{10} [\sin t + \cos t]$$

OR,

$$x = \frac{-1}{10} e^{-2t} + \frac{1}{10} e^{-3t} + \frac{1}{10} [\sin t + \cos t].$$

c) Command window

clear

clc

close all

syms t

$$x = (1/10 * \exp(-2*t)) - (1/10 * \exp(-3*t)) + (1/10 * (\sin(t) + \cos(t)))$$

$$t = 0:0.01:15$$

$$xt = \text{subs}(x, t)$$

$$xtn = \text{double}(xt)$$

plot(t, xtn)

xlabel('t')

ylabel('x')

grid on

grid minor

grid tight

3) At steady state

$$x_{t \rightarrow \infty} = x_{\text{steady state}} = 0.1 \cos t + 0.1 \sin t$$

$$0.1 \cos t + 0.1 \sin t = K \sin(t + a)$$

$$K \sin(t + a) = K \sin t \cos a + K \cos t \sin a$$

$$\therefore \text{Coefficient of } \cos t = K \sin a$$

$$\text{Coefficient of } \sin t = K \cos a$$

When squaring both sides.

$$K^2 \sin^2 a + K^2 \cos^2 a = 0.1^2 + 0.1^2$$

$$K^2 (\sin^2 a + \cos^2 a) = 0.02$$

$$K^2 = 0.02$$

$$K = \sqrt{0.02}$$

$$K = 0.1414 = \sqrt{2}/10$$

$$k \sin a = 0.1 = 1$$

$$k \cos a = 0.1$$

Remember that $\sin/\cos = \tan$

$$\tan a = 1$$

$$\tan^{-1}(1) = a$$

$$a = 45^\circ \text{ or } \frac{\pi}{4} \text{ radian}$$

state state.

$$= \frac{\sqrt{2}}{10} \sin(t + \frac{\pi}{4})$$

