Name: Ogundola Damilola Ofamade.
Department: Chemical Engineering.
Matric Number: 17/ENG01/022.
Course Code: ENG 381
Course Title: Engineering mathematics II

The dynamic model of a body in motion performing damped forces vibration is as in Equation (1)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t.$$

Given that $t=0$, $x=0.1$ and $dx/dt = 0$.

(a) Using the Auxiliary Equation Method, obtain the solution of the model in form of an expression having x as a function of t.

(b) With the aid of a MATLAB mfile program, plot the relationship between x and t for $0 \le t \le 15$ time unit using a step size of 0.01 unit end.

(c) Write the steady state solution of the model in form of $x = k\sin(t-a)$

Solution

(1)
$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t.$$

In Auxiliary form:

$$m^2 + 5m + 6 = 0.$$
$$(m+2)(m+3) = 0.$$
$$m+2=0 \quad \text{or} \quad m+3=0.$$
$$m = -2 \text{ or } -3.$$
$$x = Ae^{-2t} + Be^{-3t}$$

$P.I = \cos t$ $\quad x = C\cos t + D\sin t$.

$$dx/dt = -C\sin t + D\cos t$$
$$d^2x/dt^2 = -C\cos t - D\sin t.$$

Substituting back into the equation

$$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$$
$$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$$
$$-C\cos t + 6C\cos t - D\sin t + 6D\sin t - 5C\sin t + 5D\cos t = \cos t$$
$$5C\cos t + 5D\sin t - 5C\sin t + 5D\cos t = \cos t$$
$$5C\cos t + 5D\cos t + 5D\sin t - 5C\sin t = \cos t$$

$5C\cancel{\cos t} + 5D\cancel{\cos t} = \cancel{\cos t}$

$5C + 5D = 1 \quad \ldots (i)$

$5D\sin t - 5C\sin t = \cos t$

$5D - 5C = 0 \quad \ldots (ii)$

Using simultaneous equation.

~~$5C + 5D = 1 \quad (i)$~~
~~$-5C + 5D = 0 \quad (ii)$~~

$5C + 5D = 1 \quad \ldots (i)$

$-5C + 5D = 0 \quad \ldots (ii)$

$10D = 1$

$D = 1/10$

$5C + 5\left(\frac{1}{10}\right) = 1$

$5C + 1/2 = 1$

$5C = 1 - 1/2$

$5C = 1/2$

$C = 1/10$

$G.S = Ae^{-2t} + Be^{-3t} + \frac{1}{10}\left[\sin t + \cos t\right]$.

when $t = 0$, $x = 0.1$.

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + 1/10\left[\sin(0) + \cos(0)\right]$

$0.1 = A + B + 0 + 1/10$.

$A + B = 0 \quad \ldots (i)$.

when $t = 0$, $dx/dt = 0$.

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + 0.1(\cos t - \sin t)$.

$0 = -2A - 3B + 0.1$

$-0.1 = -2A - 3B \quad \ldots (ii)$

Remember $A + B = 0 \quad \ldots (iii)$

$A = -B \quad - (iv)$

$-0.1 = -2(-B) - 3B$

$-0.1 = 2B - 3B$

$-0.1 = -B$.

$B = 0.1$

knowing that A = -B.

A = -0.1

$\therefore x = -\frac{1}{10} - 0.1e^{-2t} + 0.1e^{-3t} + \frac{1}{10}[\sin t + \cos t]$

OR.

$x = \frac{-1}{10}e^{-2t} + \frac{1}{10}e^{-3t} + \frac{1}{10}[\sin t + \cos t]$

(2) Commandwindow

```
clear
clc
close all
syms t
x=(1/10*exp(-2*t))-(1/10*exp(-3*t))+(1/10*(sin(t)+cos(t)))
t=0:0.01:15
xt=subs(x,t)
xtn=double(xt)
Plot(t,xtn)
xlabel('t')
ylabel('x')
grid on
grind minor
grind tight.
```

(3) At steady state

$$x_{t \to \infty} = x_{\text{steady state}} = 0.1\cos t + 0.1\sin t$$

$$0.1\cos t + 0.1\sin t = k\sin(t + a)$$

$$k\sin(t+a) = k\sin t\cos a + k\cos t\sin a$$

N.B Coefficient of $\cos t = k\sin a$

,, of $\sin t = k\cos a$.

When squaring both sides.

$$k^2\sin^2 a + k^2\cos^2 a = 0.1^2 + 0.1^2$$

$$k^2(\sin^2 a + \cos^2 a) = 0.02.$$

$$k^2 = 0.02.$$ Note that $\sin^2 a + \cos^2 a = 1$

$$k = \sqrt{0.02}$$

$$k = 0.1414 \ldots = \sqrt{2}/10.$$

$$\frac{k\sin a}{k\cos a} = \frac{0.1}{0.1} = 1$$

Remember that $\sin/\cos = \tan$

$$\tan a = 1$$

$$\tan^{-1}(1) = a$$

$a = 45^0$ or $\pi/4$ radian.

Steady state

$$= \frac{\sqrt{2}}{10}\sin\left(t + \pi/4\right).$$