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ENG 381 (ENGINEERING MATHEMATICS I)

The dynamic model of a body in motion performing damped forced vibration is as in Equation (1)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that $t = 0$, $x = 0.1$ and $dx/dt = 0$

a) Using the auxillary equation method, obtain the solution of the model in form of an expression having x as a function of t.
b) With the aid of a MATLAB m-file program, plot the relationship between x and t for $0 \leq t \leq 15$ time in t using a step size of 0.01 unit end.
c) Write the steady solution of the model in form of $x = k\sin(t-a)$

Solution.

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

CF:

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$$

$$m^2 + 5m + 6 = 0$$

$$(m+2)(m+3) = 0$$

$$m_1 = -2, \quad m_2 = -3$$

$$\therefore x = Ae^{m_1 t} + Be^{m_2 t}$$

$$x = Ae^{-2t} + Be^{-3t}$$

PI: $f(t) = \cos t$

$$x = C\cos t + D\sin t$$

$$\frac{dx}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$$

Substituting values into eqn

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6[C\cos t + D\sin t] = \cos t$$

$$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$$

$$6C\cos t - C\cos t + 5D\cos t + 6D\sin t - D\sin t - 5C\sin t = \cos t$$

$$5C\cos t + 5D\cos t + 5D\sin t - 5C\sin t = \cos t$$

$$(5C + 5D)\cos t = \cos t$$

$$5C + 5D = 1 \quad \text{—— (i)}$$

equating values of LHS to RHS

$$(5D - 5C)\sin t = 0\sin t$$

$$5D - 5C = 0 \quad \text{—— (ii)}$$

$$5C + 5D = 1 \quad \text{—— (i)}$$

$$5D - 5C = 0 \quad \text{—— (ii)}$$

from eqn (ii) ; $5D = 5C$

$D = C$.

sub $D = C$ into eqn (i)

$$5C + 5(C) = 1$$

$$10C = 1$$

$$C = 1/10$$

$D = C \quad \therefore D = 1/10 \quad \therefore x = \frac{1}{10}(\cos t + \sin t)$

General solution =

$$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}(\cos t + \sin t]$$

when $t = 0$, $x = 0.1$

$$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}(\cos t + \sin t)$$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}(\cos 0 + \sin 0]$$

$e^0 = 1$, $\cos 0 = 1$, $\sin 0 = 0$

$$0.1 = A + B + \frac{1}{10}$$

$$0.1 - \frac{1}{10} = A + B$$

$0 = A + B \quad \therefore \quad A = -B$

when $t = 0$, $\frac{dx}{dt} = 0$

$$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}(\cos t + \sin t)$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + \frac{1}{10}(-\sin t + \cos t)$$

$$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} + \frac{1}{10}(-\sin 0 + \cos 0)$$

$e^0 = 1$, $\cos 0 = 1$, $\sin 0 = 0$

$$0 = -2A - 3B + \frac{1}{10}$$

$$0 - \frac{1}{10} = -2A - 3B$$

$-0.1 = -2A - 3B$ —— (ii)

recall $A = -B$

$-0.1 = -2(-B) - 3B$

$-0.1 = 2B - 3B$

$+0.1 = +B$

$B = 1/10$; $A = -1/10$

$\therefore A = -1/10$, $B = 1/10$.

$$x = \frac{-e^{-2t}}{10} + \frac{e^{-3t}}{10} + \frac{1}{10}(\cos t + \sin t)$$

$$x = -0.1e^{-2t} + 0.1e^{-3t} + 0.1(\cos t + \sin t)$$

At steady state;

$$x_{t \to \infty} = x_{\text{steady state}} = 0.1\cos t + 0.1\sin t$$

$0.1\cos t + 0.1\sin t = k\sin(t + a)$

$k\sin(t + a) = k\sin t\cos a + k\cos t\sin a$

N.B coefficient of $\cos t = k\sin a$

" " of $\sin t = k\cos a$

when squaring both sides

$k^2 \sin^2 a + k^2 \cos^2 a = 0.1^2 + 0.1^2$

$k^2(\sin^2 a + \cos^2 a) = 0.02$

$k^2 = 0.02$ bcos $\sin^2 a + \cos^2 a = 1$

$k = \sqrt{0.02}$

$k = 0.1414 = \frac{\sqrt{2}}{10}$

$\frac{k \sin a}{k \cos a} = \frac{0.1}{0.1} = 1$

recall $\frac{\sin a}{\cos a} = \tan a$

$\tan a = 1$

$\tan^{-1}(1) = a$

$a = 45^\circ$ or $\frac{\pi}{4}$ radian

steady state

$= \frac{\sqrt{2}}{10} \sin(t + \pi/4)$

MATLAB:

Command window

```
clear
clc
close all
syms t
x = (1/10 * exp(-2*t)) - (1/10 * exp(-3*t)) + (1/10 * (sin(t) + cos(t)))
t = 0:0.01:15
xt = subs(x, t)
xtn = double(xt)
Plot (t, xtn)
x label ('t')
y label ('x')
grid on
grid minor
grid light
```

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