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ENG 381
Mechanical Engineering

The dynamic model of a body in motion performing damped forces vibration is a sin equation (i)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that $t = 0$, $x = 0.1$ and $dx/dt = 0$

a) Using the auxilary equation method, obtain the solution of the model information of an expression having x as a function of t.

(b) With the aid of mathlab mfile program, plot the relationship between x and t for $0 \leq t \leq 15$ time int using a step size of 0.01 unit end.

(c) Write the steady state solution of the model inform of $x = k\sin(t-a)$

SOLUTION

(1) $$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

In Auxilary form

$m^2 + 5m + 6 = 0$

$m^2 + 2m + 3m + 6 = 0$

$(m^2 + 2m)(3m + 6)$

$m(m+2)\,3(m+2)$

$m = -2$, $m = -3$

$x = Ae^{-2t} + Be^{-3t}$

$PI = \cos t$

$x = C\cos t + D\sin t$

$dx/dt = -C\sin t + D\cos t$

$d^2x/dt^2 = -C\cos t - D\sin t$

Substituting back into equation

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$-C\cos t + 6C\cos t - D\sin t + 6D\sin t - 5C\sin t + 5D\cos t = \cos t$

$5C\cos t + 5D\sin t - 5C\sin t + 5D\cos t = \cos t$

$5C\cos t + 5D\cos t + 5D\sin t - 5C\sin t = \cos t$

$5C\cos t + 5D\cos t = \cos t$

$5C + 5D = 1 \quad --- (i)$

$5D\sin t - 5C = 0$

$5D - 5C = 0 \quad --- (ii)$

Using simutaneous equation

$5C + 5D = 1$

$-5C + 5D = 0$

$10D = 1$

$D = 1/10$

$5C + 5(1/10) = 1$

$5C + 1/2 = 1$

$5C = 1 - 1/2$

$5C = 1/2$

$C = 1/10$

$G.S = Ae^{-2t} + Be^{-3t} + 1/10[\sin t + \cos t]$

When $t = 0$, $x = 0.1$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + 1/10[\sin(0) + \cos(0)]$

$0.1 = A + B + 0 + 1/10$

$A + B = 0 \quad - (i)$

When $t = 0$, $dx/dt = 0$

$dx/dt = -2Ae^{-2t} - 3Be^{-3t} + 0.1(\cos t - \sin t)$

$0 = -2A - 3B + 0.1$

$-0.1 = -2A - 3B \quad - iii)$

Rember $A + B = 0$

$A = -B$ (iv)

$-0.1 = -2(-B) - 3B$

$-0.1 = 2B - 3B$

$-0.1 = -B$

$B = 0.1$

knowing that $A = -B$

$A = -0.1$

$\therefore x = -0.1e^{-2t} + 0.1e^{-3t} + 1/10\,[\sin t + \cos t]$ OR

$$x = \frac{-1}{10}e^{-2t} + \frac{1}{10}e^{-3t} + \frac{1}{10}[\sin t + \cos t]$$

2 Command window

```
Clear
clc
close all
Syms t
x = (1/10 *exp(-2*t) - (1/10*exp(-3*t)) +(1/10 (sint) +cost(t))
t = 0:0.01:15
xt = subs (x,t)
xtn = double (xt)
Plot (t,xtn)
xlabel ('t')
ylabel ('x')
grid on
grid minor
grid right
```

3 At steady state

$x_{t\to\infty} = x_{\text{steady state}} = 0.1\cos t + 0.1\sin t$

$0.1\cos t + 0.1\sin t = k\sin(t+a)$

$k\sin(t+a) = k\sin t\cos a + k\cos t\sin a$

NB coefficent of cos t = k sin a

" " " sin t = k cos a

When squaring both sides

$k^2 \sin^2 a + k^2 \cos^2 a = 0.1^2 + 0.1^2$

$k^2(\sin^2 a + \cos^2 a) = 0.02$

$k^2 = 0.02$

$k = \sqrt{0.02}$

$k = 0.144 = \sqrt{2}/10$

$\frac{k \sin a}{k \cos a} = \frac{0.1}{0.1} = 1$

Rember that sin/cos = tan

$\tan a = 1$

$\tan^{-1}(1) = a$

$a = 45°$ or $\pi/4$ radian

Steady state

$= \frac{\sqrt{2}}{10} \sin(t + \pi/a)$