The dynamic model of a body in motion performing damped forces vibration is as in equation (i)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t \cdot$$

Given that $t=0$, $x=0.1$ and $dx/dt=0$.

(a) Using the Auxiliary Equation Method, obtain the solution of the model in form of an expression having $x$ as a function of $t$.

(b) With the aid of a MATLAB mfile program, plot the relationship between $x$ and $t$ for $0 \le t \le 15$ time unit using a step size of 0.01 unit end.

(c) Write the steady state solution of the model in form of $x = k\sin(t-a)$

Solution.

(i) $$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t \cdot$$

In Auxiliary form:

$$m^2 + 5m + 6 = 0 \cdot$$

$$(m+2)(m+3) = 0 \cdot$$

$$m+2=0 \quad \text{or} \quad m+3=0 \cdot$$

$$m = -2 \text{ or } -3 \cdot$$

$$x = Ae^{-2t} + Be^{-3t}$$

P.I $= \cos t$ $\quad -C\cos t + D\sin t \cdot$

$$dx/dt = -C\sin t + D\cos t$$

$$d^2x/dt^2 = -C\cos t - D\sin t \cdot$$

Substituting back into the equation

$$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$$

$$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$$

$$-C\cos t + 6C\cos t - D\sin t + 6D\sin t - 5C\sin t + 5D\cos t = \cos t$$

$$5C\cos t + 5D\sin t - 5C\sin t + 5D\cos t = \cos t$$

$$5C\cos t + 5D\cos t + 5D\sin t - 5C\sin t = \cos t$$

$5C\cos t + 5D\cos t = \cos t$

$5C + 5D = 1 \quad \ldots (i)$

$5D\sin t - 5C\sin t = \cos t$

$5D - 5C = 0 \quad \ldots (ii)$

Using simultaneous equation:

$5C + 5D = 1 \quad \ldots (i)$

$-5C + 5D = 0 \quad \ldots (ii)$

$10D = 1$

$D = 1/10$

$5C + 5\left(\frac{1}{10}\right) = 1$

$5C + 1/2 = 1$

$5C = 1 - 1/2$

$5C = 1/2$

$C = 1/10$

$G.S = Ae^{-2t} + Be^{-3t} + \frac{1}{10}[\sin t + \cos t]$

When $t = 0$, $x = 0.1$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + 1/10[\sin(0) + \cos(0)]$

$0.1 = A + B + 0 + 1/10$

$A + B = 0 \quad \ldots (i)$

when $t = 0$, $dx/dt = 0$

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + 0.1(\cos t - \sin t)$

$0 = -2A - 3B + 0.1$

$-0.1 = -2A - 3B \quad \ldots (ii)$

Rembember $A + B = 0 \quad \ldots (iii)$

$A = -B \quad \ldots (iv)$

$-0.1 = -2(-B) - 3B$

$-0.1 = 2B - 3B$

$-0.1 = -B$

$B = 0.1$

knowing that $A = -B$.

$A = -0.1$

$$\therefore x = \frac{-1}{10} - 0.1e^{-2t} + 0.1e^{-3t} + \frac{1}{10}\left[\sin t + \cos t\right]$$

OR.

$$x = \frac{-1}{10}e^{-2t} + \frac{1}{10}e^{-3t} + \frac{1}{10}\left[\sin t + \cos t\right]$$

(2)

```
Commandwindow
clear
clc
close all
syms t
x=(1/10*exp(-2*t))-(1/10*exp(-3*t))+(1/10*(sin(t)+cos(t)))
t=0:0.01:15
xt=subs(x,t)
xtn=double(xt)
plot(t,xtn)
xlabel('t')
ylabel('x')
grid on
grind minor
grind right.
```

(b) At steady state

$x_{t\to\infty} = x_{steady\ state} = 0.1\cos t + 0.1\sin t$

$0.1\cos t + 0.1\sin t = k\sin(t+a)$

$k\sin(t+a) = k\sin t\cos a + k\cos t\sin a$

NB Coeffcent of cost = $k\sin a$

.1) of $\sin t = k\cos a$.

When squaring both sides.

$k^2\sin^2 a + k^2\cos^2 a = 0.1^2 + 0.1^2$

$k^2(\sin^2 a + \cos^2 a) = 0.02$.

$k^2 = 0.02$.

Note that $\sin^2 a + \cos^2 a = 1$

$k = \sqrt{0.02}$

$k = 0.1414 = \sqrt{2}/10$.

$\frac{k\sin a}{k\cos a} = \frac{0.1}{0.1} = 1$

Remember that $\sin/\cos = \tan$

$\tan a = 1$

$\tan^{-1}(1) = a$

$a = 45^0$ or $\pi/4$ radien.

steady state

$-\frac{\sqrt{2}}{10}\sin(t + \pi/a)$.