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Petroleum Engineering

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Assignment 1 - Answer

① Auxiliary Method

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \text{Cost} \quad \text{--- eqn ①}$$

from ①

$$m^2 + 5m + 6 = 0$$

$$m^2 + 3m + 2m + 6 = 0$$

$$m(m+3) + 2(m+3) = 0$$

$$(m+3)(m+2) = 0$$

$$m = -3 \text{ or } m = -2$$

Complimentary function $\Rightarrow \mathcal{L}^{-1} A e^{-3t} + B e^{-2t}$

Particular Integral

$$\mathcal{L}^{-1} C \cos t + D \sin t \quad \text{--- ②}$$

$$\frac{dx}{dt} = -C \sin t + D \cos t \quad \text{--- ③}$$

$$\frac{d^2x}{dt^2} = -C \cos t + (-D \sin t) \quad \text{--- ④}$$

Put ②, ③, ④ in eqn ①

$$-C_{\text{cost}} - D_{\text{int}} + 5(-C_{\text{int}} + D_{\text{cost}}) + b(C_{\text{cost}} + D_{\text{int}}) = C_{\text{cost}}$$

$$-C_{\text{cost}} - D_{\text{int}} - 5C_{\text{int}} + 5D_{\text{cost}} + bC_{\text{cost}} + bD_{\text{int}} = C_{\text{cost}}$$

Taking co-efficient of Cost

$$-C + 5D + bC = 1$$

$$5D + 5C = 1 \quad \text{--- (B)}$$

Taking co-efficient of Int

$$-D - 5C + bD = 0$$

$$5D - 5C = 0 \quad \text{--- (A)}$$

from equ (A)

$$5D = 1 - 5C \quad \text{--- (P)}$$

Put (P) in (B)

$$(1 - 5C) - 5C = 0$$

$$1 - 5C - 5C = 0$$

$$1 - 10C = 0$$

$$1 = 10C$$

$$C = \frac{1}{10} = 0.1$$

Put C in (P)

$$5D = 1 - 5\left(\frac{1}{10}\right)$$

$$5D = 1 - \frac{1}{2}$$

$$5D = \frac{1}{2}$$

$$D = \frac{1}{10} = 0.1$$

$$PI \Rightarrow x = 0.1C_{\text{cost}} + 0.1S_{\text{int}}$$

Particular solution

$$x = Ae^{-3t} + Be^{-2t} + 0.1 \cos t + 0.1 \sin t$$

When $t=0$, $x=0.1$

$$0.1 = A + B + 0.1$$

$$0 = A + B \quad \text{--- (8)}$$

$$A = -B$$

When $t=0$, $\frac{dx}{dt} = 0$

$$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} - 0.1 \sin t + 0.1 \cos t$$

$$0 = -3A - 2B - 0 + 0.1$$

$$0 = -3A - 2B + 0.1$$

$$0 = -3(-B) - 2B + 0.1$$

$$0 = 3B - 2B + 0.1$$

$$0 = B + 0.1$$

$$B = -0.1$$

Put B in A

$$A = -(-0.1)$$

$$= 0.1$$

General solution

$$x = 0.1e^{-3t} + (-0.1)e^{-2t} + 0.1 \cos t + 0.1 \sin t$$

$$x = 0.1e^{-3t} - 0.1e^{-2t} + 0.1 \cos t + 0.1 \sin t$$

④ Mfile

Command window

clc

clear all

close all

Syms x, t

t = 0:0.01:15

$x = 0.1 * \exp(-3 * t) - 0.1 * \exp(-2 * t) + 0.1 * \cos(t) + 0.1 * \sin(t)$

$x_n = \text{subs}(x)$

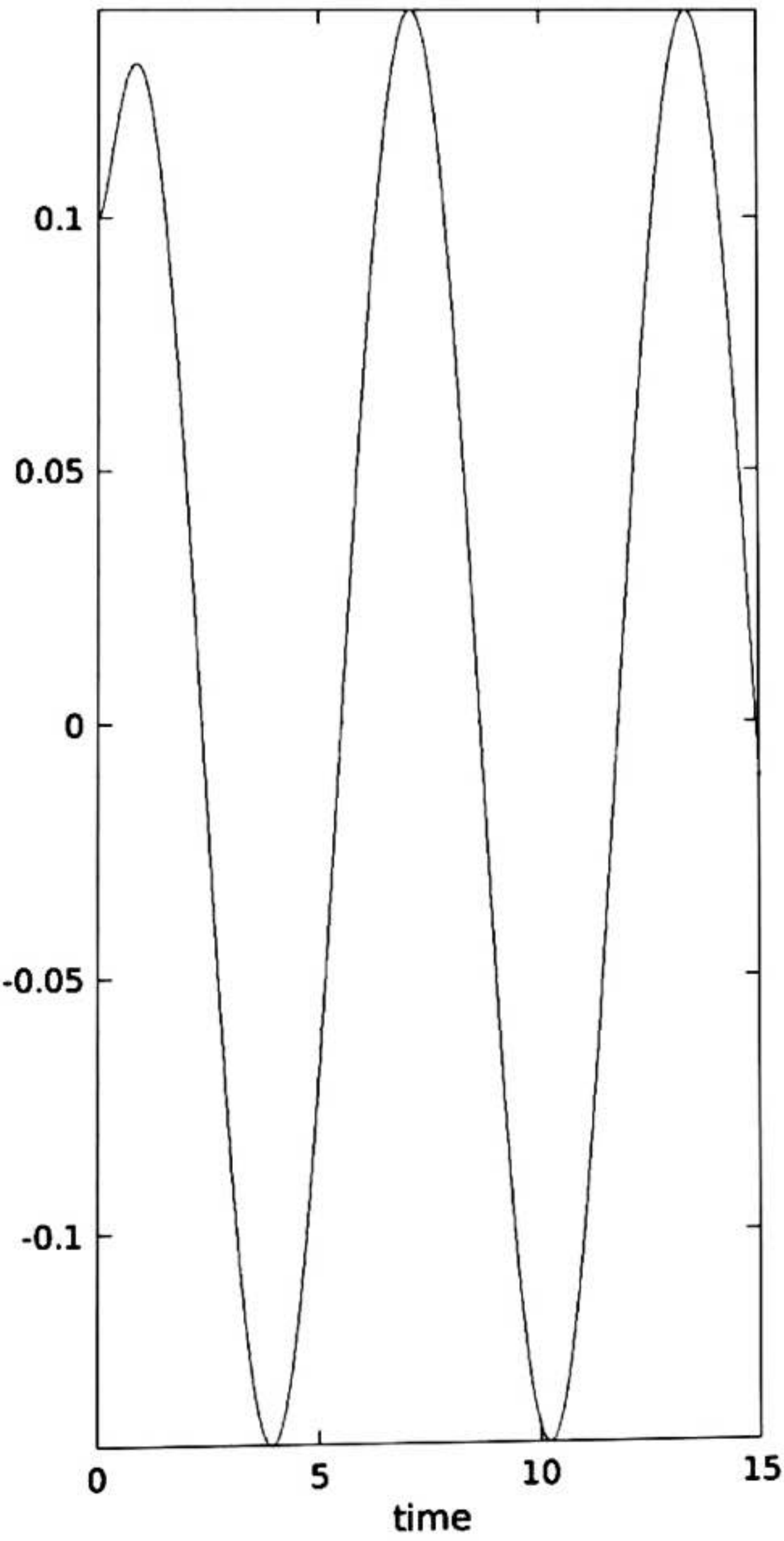
Plot (t, x_n)

xlabel('Time')

grid on

grid minor

axis tight



$$\textcircled{a} 0.1 \cos t + 0.1 \sin t = K \sin(t + \alpha) \text{ at steady state}$$

$$0.1 \cos t + 0.1 \sin t = K \sin t \cos \alpha + K \cos t \sin \alpha$$

Comparing Co-efficient

Co-efficient of $\cos t$

$$0.1 = K \sin \alpha$$

Co-efficient of $\sin t$

$$0.1 = K \cos \alpha$$

Assume

$$K^2 \sin^2 \alpha + K^2 \cos^2 \alpha = 0.1^2 + 0.1^2$$

$$K^2 (\sin^2 \alpha + \cos^2 \alpha) = 0.02$$

$$K^2 = 0.02 \dots \dots \dots (\sin^2 \alpha + \cos^2 \alpha = 1)$$

$$K^2 = \frac{2}{100}$$

$$K = \frac{\sqrt{2}}{\sqrt{100}}$$

$$\frac{K \sin \alpha}{K \cos \alpha} = \frac{0.1}{0.1} = 1$$

$$\tan \alpha = 1$$

$$\alpha = \tan^{-1}(1)$$

$$\alpha = \pi/4 \text{ or } 45^\circ$$

$$x \text{ at steady state; } \frac{\sqrt{2}}{10} \sin \left[\frac{\pi}{4} + t \right]$$