

11/10/19 FIDE-AKWUOBI ANTHONY CHIZALU ENG 381 MECHANICAL ENGINEERING 300 LVL 17/eng 06/037

$$(1) \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$\therefore m^2 + 5m + 6 = 0$$

$$m^2 + 3m + 2m + 6 = 0$$

$$m(m+3) + 2(m+3) = 0$$

$$m+2=0 \text{ or } m+3=0$$

$$\therefore m = -2 \text{ or } m = -3$$

$$\therefore \text{C.F.} = Ae^{-2t} + Be^{-3t}$$

$$P.I = \cos t$$

$$x = C\cos t + D\sin t$$

$$\frac{dx}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$$

$$\Rightarrow (-C\cos t - D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$$

$$= -C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$$

$$(-C\cos t + 5D\cos t + 6C\cos t) + (-D\sin t - 5C\sin t + 6D\sin t) = \cos t$$

$$5C\cos t + 5D\cos t = \cos t \quad \text{--- (i)}$$

$$5D\sin t - 5C\sin t = 0 \quad \text{--- (ii)}$$

$$\therefore 5C + 5D = 1$$

$$-5C + 5D = 0 \quad \text{solving simultaneously}$$

$$\therefore 10D = 1$$

$$\therefore D = \frac{1}{10}$$

$$5C + 5D = 1$$

$$\therefore 5C + 5\left(\frac{1}{10}\right) = 1$$

$$\therefore 5C + \frac{1}{2} = 1$$

$$\therefore 5C = \frac{1}{2} \quad \therefore C = \frac{1}{10}$$

$$\therefore P.I = \frac{1}{10} (\cos t + \sin t)$$

$$x = C.F + P.I$$

$$\therefore x = Ae^{-2t} + Be^{-3t} + \frac{1}{10} (\cos t + \sin t)$$

$$\text{when } t = 0 \quad \therefore x = 0.1 \text{ and } \frac{dx}{dt} = 0$$

$$\therefore 0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10} (\cos 0 + \sin 0)$$

$$0.1 = A + B + 0.1$$

$$A + B = 0.1 - 0.1$$

$$\therefore A + B = 0 \quad \text{--- eqn (3)}$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\therefore \frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + \frac{1}{10} (\cos t + \sin t)$$

$$0 = -2Ae^{-2(0)} - \cancel{3B} 3Be^{-3(0)} + \frac{1}{10} (1)$$

$$= -2A - 3B + 0.1$$

$$\therefore 3B + 2A = 0.1 \quad \text{--- eqn (4)}$$

$$\text{Recall } A + B = 0 \quad \text{--- (3)}$$

$$\therefore A = -B \quad \text{--- (5)}$$

$\therefore$  substitute in eqn (4)

$$\therefore 3B + 2(-B) = 0.1$$

$$\therefore B = 0.1$$

$$\therefore \text{Since } A = -B \quad \therefore A = -0.1$$

$$\therefore x = -0.1e^{-2t} + 0.1e^{-3t} + \frac{1}{10} [\sin t + \cos t]$$

2. command window

clear

clc

close all

syms t

$$x = [0.1 * \exp(-2*t) - (0.1 * \exp(-3*t)) + (0.1) * [\sin t + \cos t]]$$

$$t = 0 : 0.01 : 15$$

~~xxxxxx~~

Plot (t,x)

x label ('t')

y label ('x')

grid on

grid minor

grid right

3. At steady state

$$x = 0.1 \cos t + 0.1 \sin t$$

$$0.1 \cos t + 0.1 \sin t = k \sin(t + \alpha)$$

$$k \sin(t + \alpha) = k \sin t \cos \alpha + k \cos t \sin \alpha$$

Square both sides-

$$\therefore k^2 \sin^2 \alpha + k^2 \cos^2 \alpha = 0.1^2 + 0.1^2$$

$$k^2 (\sin^2 \alpha + \cos^2 \alpha) = 0.02$$

$$k^2 = 0.02$$

$$k = \sqrt{0.02}$$

$$k = 0.144 = \frac{\sqrt{2}}{10}$$

$$\therefore \frac{k \sin \alpha}{k \cos \alpha} = \frac{0.1}{0.1} = 1 \quad \dots \text{Remember that } \sin / \cos = \tan$$

$$\therefore \tan \alpha = 1$$

$$\therefore \tan^{-1}(1) = \alpha$$

$$\alpha = 45^\circ \text{ or } \frac{\pi}{4} \text{ in radians.}$$

$$\therefore \text{Steady State} = \frac{\sqrt{2}}{10} \sin\left(t + \frac{\pi}{4}\right) \text{ or } \frac{\sqrt{2}}{10} \sin(t + 45^\circ)$$

