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17/ENG01/074
ELECT/ELECT. ENG 351 ASSIGNMENT

- The dynamic model of a body in motion Describing damped forced vibrations is as in eqn ①

$$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$$

Given that when $t=0$, $x=0.1$ and $\frac{dx}{dt} = 0$

② Using Auxiliary Equation method, Obtain the Solution of the model in the form of an expression having x as a function of t

③ With the aid of MATLAB m-file Program, plot the relationship between x and t for $0 \leq t \leq 15$ time unit using a step size of unit and

④ Write the steady-State Solution of the model in form of $x = A\sin(t+\alpha)$.

Solu.

② $\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$

Auxiliary Eqn: $m^2 + 5m + 6 = 0$

$m^2 + 2m + 3m + 6 = 0$

$m(m+2) + 3(m+2) = 0$

$(m+3)(m+2) = 0$

$m = -3$ or -2

Complementary function: $x = Ae^{-3x} + Be^{-2x}$

Particular integral. Assume $x = C\cos t + D\sin t$

$\therefore \frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

Putting the value of d^2x/dt^2 and dx/dt in the equation

$\therefore -(C\cos t - D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

Comparing Co-efficients

Co-efficient of $\cos t$: $-C + 5D + 6C = 1$ ---①

$\sin t$: $-D - 5C + 6D = 0$ ---②

From ① : $5C + 5D = 1$ ---③

② , $5D - 5C = 0$ ---④

③ , $5D = 1 - 5C$ ---⑤

Subst. ⑤ into ④

$1 - 5C - 5C = 0$

$1 - 10C = 0$

$C = 1/10 = 0.1$

$\therefore 5D = 1 - 5C$

Subst value of C in ⑤

$5D = 1 - 5(0.1)$

$5D = 0.5$

$D = 0.5/5 = 0.1$

$\therefore D = 0.1$

Particular Solution $x = Ae^{-3t} + Be^{-2t} + 0.1\sin t + 0.1\cos t$

when $t = 0$; $x = 0.1$, $\frac{dx}{dt} = 0$

$0.1 = A + B + 0.1$

$A + B = 0$ ---⑥

$\frac{dy}{dx} = -3Ae^{-3t} - 2Be^{-2t} + 0.1\sin t + 0.1\cos t$

$0 = -3A - 2B + 0.1$

From ⑥ $B = -A$

$0 = -3A - 2(-A) + 0.1$

$0 = -A + 0.1$

$A = 0.1$

and $B = -A$

$B = -0.1$

General Solution $= 0.1e^{-3t} - 0.1e^{-2t} + 0.1\sin t + 0.1\cos t$ //

2) - Command window
- Clear
- clc
- Closeall
- Syms k t
- K = 0.1*exp(-3*t) - 0.1*exp(-2*t) + 0.1*cos(t) + 0.1*sin(t)
- t = 0, 0.01 : 15
- Kn = Subs(K)
- Plot (t, Kn)
- Xlabel ('time')
- Grid on
- grid minor
- axis tight

C. $0.1\cos t + 0.1\sin t = K\sin(t+\alpha)$ as steady flow

$0.1\cos t + 0.1\sin t = K\sin t\cos\alpha + K\sin\alpha\cos t$

Comparing Coefficients

$\cos t : 0.1 = K\sin\alpha$

$\sin t : 0.1 = K\cos\alpha$

Square $K\sin\alpha$ and $K\cos\alpha$ and equate it to the addition

$K^2\sin^2\alpha + K^2\cos^2\alpha = 0.1 + 0.1$

$K^2(\sin^2\alpha + \cos^2\alpha) = 0.2$, Since $(\sin^2\alpha + \cos^2\alpha = 1)$

$\therefore \quad K^2 = 0.2$

$K^2 = \frac{2}{100}$

$K = \frac{\sqrt{2}}{10}$

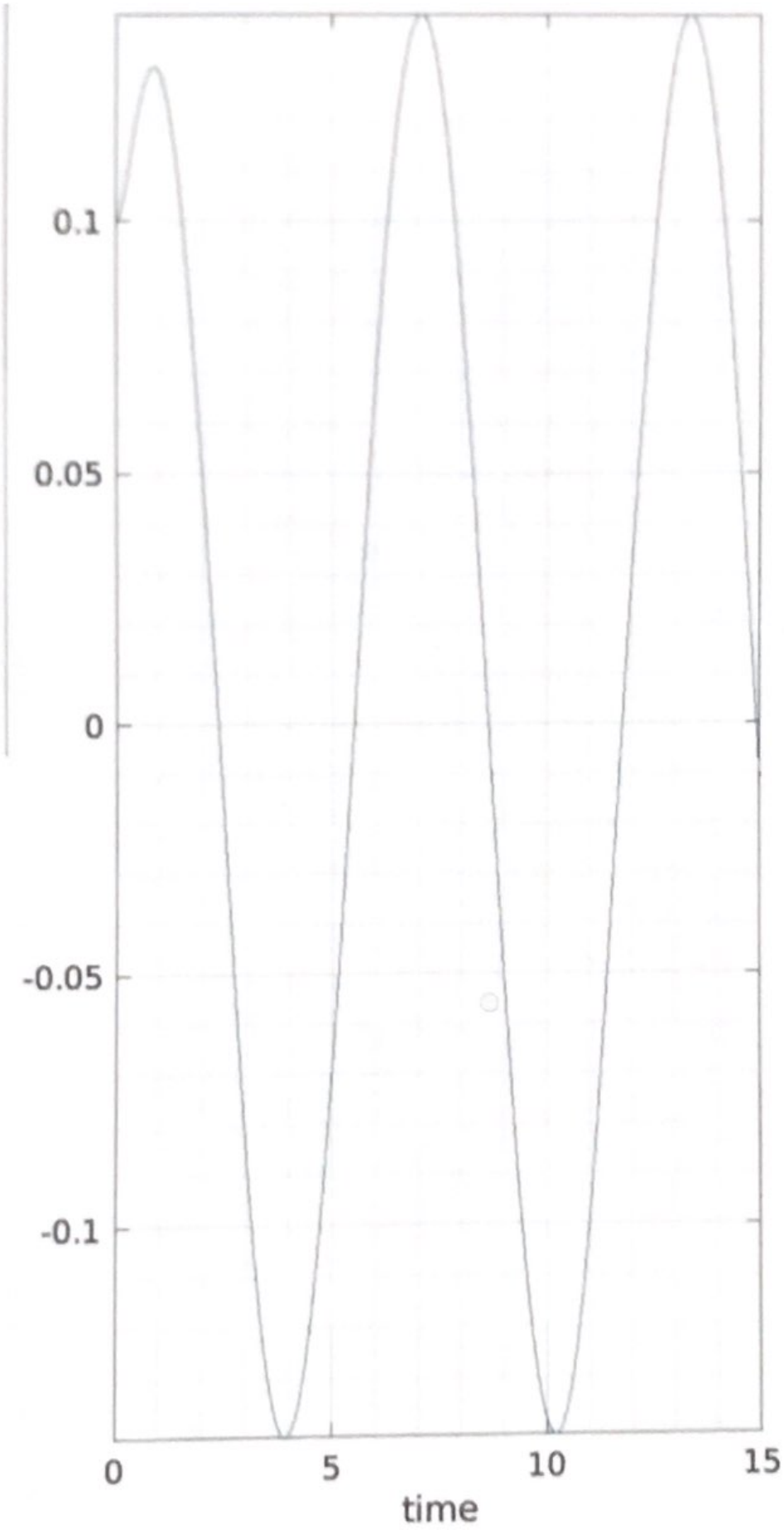
$\frac{K\sin\alpha}{K\cos\alpha} = \frac{0.1}{0.1}$

$\tan\alpha = 1$

$\alpha = \tan^{-1}(1)$

$\alpha = 45^\circ$ or $\pi/4$

$\therefore$ K steady state; $K_{ss} = \frac{\sqrt{2}}{10}\sin\left(\frac{\pi}{4} + t\right)$


0.1
0.05
0
-0.05
-0.1
0
5
10
15
time