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MATRIC NO: 18/ENGO1/1214

(1) (a) Show that the limit of the function given in Equation (1.1) as x approaches

$$0 \text{ is } \frac{a}{b} \quad f(x) = \frac{\sin ax}{bx}$$

Solution

$$f(x) = \frac{\sin ax}{bx}$$

$$x \rightarrow 0$$

$$f(x) = \frac{\sin ax \cdot 0}{b \cdot 0} = \frac{0}{0} \text{ - Undetermined}$$

Differentiate $f(x)$

$$f'(x) = \frac{a \cos ax}{b}$$

$$x \rightarrow 0$$

$$f'(x) = \frac{a \cos ax}{b} = \frac{a \cos 0}{b}$$

$$= \frac{a \times 1}{b} = \frac{a}{b}$$

∴

(b) The model of a system has been developed to be as given in Equation (1.2)

$$f(x) = 8x^{-2.1}$$

Given that $\delta = 0.1$ and $\Delta \delta = 0.01$, demonstrate, in tabular form, that the limit of the model as $x \rightarrow b$ is equal to 9

Solution

$$\text{for } \delta = 0.1 \text{ we form } b \text{ (LHS)} \quad b - 0.1 = 8.9$$

$$\text{for } \delta = 0.1 \text{ we have } b + 0.1 = 9.1$$

$$\text{Where } a = b \quad L =$$

$L-E$	$a-\delta$	a	$a+\delta$	$L+E$
8.50	5.9	b	6.1	9.50
8.55	5.91		6.09	9.45
8.60	5.92		6.08	9.40
8.65	5.93		6.07	9.35
8.70	5.94		6.06	9.30
8.75	5.95		6.05	9.25
8.80	5.96		6.04	9.20
8.85	5.97		6.03	9.15
8.90	5.98		6.02	9.10
8.95	5.99		6.01	9.05
9.00	6.00		6.00	9

(C) Show whether the function given in Equation (1.3) is continuous on the interval $[-5, 5]$. $f(x) = (25 - x^2)^{1/2}$

Solution:

$$f(x) = (25 - x^2)^{1/2}$$

When $x = -5$

$$\begin{aligned} f(x) &= (25 - (-5)^2)^{1/2} \\ &= (25 - 25)^{1/2} = \sqrt{0} \\ &= 0 \end{aligned}$$

$$f(x) = (25 - x^2)^{1/2}$$

when $x = -4$

$$\begin{aligned} f(x) &= (25 - (-4)^2)^{1/2} \\ &= (25 - 16)^{1/2} = \sqrt{9} \\ &= 3 \end{aligned}$$

$$f(x) = (25 - x^2)^{1/2}$$

When $x = -3$

$$\begin{aligned} f(x) &= (25 - (-3)^2)^{1/2} \\ &= (25 - 9)^{1/2} = 16^{1/2} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

$$f(x) = (25 - x^2)^{1/2}$$

$$\text{When } x = -2$$

$$f(x) = (25 - (-2)^2)^{1/2}$$

$$= 25 - 4 = 21^{1/2}$$

$$= \sqrt{21} = 4.58$$

$$f(x) = (25 - x^2)^{1/2}$$

$$\text{When } x = -1$$

$$f(x) = (25 - (-1)^2)^{1/2}$$

$$= (25 - 1)^{1/2} = 24^{1/2}$$

$$= \sqrt{24}$$

$$= 2\sqrt{6}$$

$$= 4.899$$

$$= \cancel{5.0}$$

$$f(x) = (25 - x^2)^{1/2}$$

$$\text{When } x = 0$$

$$f(x) = (25 - (0)^2)^{1/2}$$

$$= 25^{1/2} = \sqrt{25}$$

$$= 5$$

$$f(x) = (25 - x^2)^{1/2}$$

$$\text{When } x = 1$$

$$f(x) = (25 - (1)^2)^{1/2}$$

$$= (25 - 1)^{1/2} = 24^{1/2}$$

$$= \sqrt{24}$$

$$= 2\sqrt{6} = 4.899$$

$$f(x) = (25 - x^2)^{1/2}$$

$$x = 2$$

$$f(x) = (25 - 2^2)^{1/2}$$

$$= (25 - 4)^{1/2} = 21^{1/2} = \sqrt{21}$$

$$= 4.58$$

$$f(x) = (25 - x^2)^{1/2}$$

When $x = 3$

$$\begin{aligned} f(3) &= (25 - 3^2)^{1/2} \\ &= (25 - 9)^{1/2} \\ &= 16^{1/2} = \sqrt{16} \\ &= 4 \end{aligned}$$

$$f(x) = (25 - x^2)^{1/2}$$

When $x = 4$

$$\begin{aligned} f(4) &= (25 - 4^2)^{1/2} \\ &= (25 - 16)^{1/2} \\ &= 9^{1/2} = \sqrt{9} \\ &= 3 \end{aligned}$$

$$f(x) = (25 - x^2)^{1/2}$$

When $x = 5$

$$\begin{aligned} f(5) &= (25 - 5^2)^{1/2} \\ &= (25 - 25)^{1/2} \\ &= 0^{1/2} = \sqrt{0} \\ &= 0 \end{aligned}$$