The dynamic model of a body in motion performing damped forces vibration is as in Equation (i)

$$\frac{d^2x}{dx^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that $t=0$, $x=0.1$ and $dx/dt = 0$

a) Using the Auxillary Equation method, obtain the solution of the model in form of an expression having x as a function of t.

b) With the aid of matlab m file program, plot the relationship between x and t for $0 \leq t \leq 15$ time unit using a step size of 0.01 unit and;

c) write the steady state solution of the model in form of $x = K\sin(t-a)$

Solution.

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Auxillary form.

$$m^2 + 5m + 6 = 0$$

$$(m+2)(m+3) = 0$$

$$m+2=0 \text{ or } m+3=0$$

$$m = -2 \text{ or } -3$$

$$x = Ae^{-2t} + Be^{-3t}$$

$$P.I = \cos t$$

$$x = -C\cos t + D\sin t$$

$$dx/dt = -C\sin t + D\cos t$$

$$d^2x/dt^2 = -C\cos t - D\sin t.$$

Substituting back into the equation.

$$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$$

$$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$$

$$-C\cos t + 6C\cos t - D\sin t + 6D\sin t - 5C\sin t + 5D\cos t = \cos t$$

$$5C\cos t + 5D\sin t - 5C\sin t + 5D\cos t = \cos t$$

$$5C\cos t + 5D\cos t + 5D\sin t - 5C\sin t = \cos t$$

$5C\cos t + 5D\cos t = \cos t$

$5C + 5D = 1$ ---(i)

$5D\sin t - 5C\sin t = \cos t$

$5D - 5C = 0$ ---(ii)

Using simultaneous equation.

$5C + 5D = 1$ — (i)

$-5C + 5D = 0$ — (ii)

$10D = 1$

$D = 1/10$

$5C + 5D = 1$

$D = 1/10$

$5C + 5(1/10) = 1$

$5C + 1/2 = 1$

$5C = 1 - 1/2$

$5C = 1/2$

$C = 1/10$

General solution $= Ae^{-2t} + Be^{-3t} + \frac{1}{10}[\sin t + \cos t]$

when $t = 0$, $x = 0.1$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + 1/10[\sin(0) + \cos(0)]$

$0.1 = A + B + 0 + 1/10$

$A + B = 0$ ---(i)

when $t = 0$, $dx/dt = 0$

$dx/dt = -2Ae^{-2t} - 3Be^{-3t} + 0.1(\cos t - \sin t)$

$0 = -2A - 3B + 0.1$

$-0.1 = -2A - 3B$ --- (ii)

Remember that $A + B = 0$ (iii)

$A = -B$ ... (iv)

$-0.1 = -2(-B) - 3B$

$-0.1 = 2B - 3B$

$-0.1 = -B$

$B = 0.1$

Knowing that $A = -B$

$A = -0.1$.

$\therefore x = -0.1e^{-2t} + 0.1e^{-3t} + \frac{1}{10}[\sin t + \cos t]$

OR

$$x = \frac{-1}{10} e^{-2t} + \frac{1}{10} e^{-3t} + \frac{1}{10} [\sin t + \cos t].$$

c) Commandwindow

```
clear
clc
close all
Syms t
x=(1/10*exp(-2*t))-(1/10*exp(-3*t))+(1/10*(sin(t)+cos(t)))
t=0:0.01:15
xt=subs(x,t)
xtn=double(xt)
plot(t, xtn)
xlabel('t')
ylabel('x')
grid on
grid minor
grid tight
```

3) At steady state

$x_{t \to \infty} = x_{steady\,state} = 0.1\cos t + 0.1\sin t.$

$0.1\cos t + 0.1\sin t = K\sin(t+a)$

$K\sin(t+a) = K\sin t\cos a + K\cos t\sin a$

$\therefore$ Coefficient of $\cos t = K\sin a$

coefficient of $\sin t = K\cos a$.

when squaring both sides.

$K^2\sin^2 a + K^2\cos^2 a = 0.1^2 + 0.1^2$

$K^2(\sin^2 a + \cos^2 a) = 0.02$

$K^2 = 0.02.$

$K = \sqrt{0.02}$

$K = 0.1414 = \sqrt{2}/10$ //

$\frac{K\sin a}{K\cos a} = \frac{0.1}{0.1} = 1$

Remember that $\sin/\cos = \tan$

$\tan a = 1$

$\tan^{-1}(1) = a$

$a = 45°$ or $\pi/4$ radian

steate state.

$= \frac{\sqrt{2}}{10} \sin(t + \pi/a)$