

ENQ 281: ENGINEERING MATHEMATICS

10) Show that the limit of the function given in eq(1.1) as x approaches 0 is a/b . $f(x) = \frac{\sin ax}{bx}$

solution

$$\lim_{x \rightarrow 0} f(x) = \frac{\sin ax}{bx}$$

Using direct substitution,

$$\frac{\sin(a \cdot 0)}{b(0)} = \text{indeterminate}$$

hence, we use L'Hopital's rule

$$\lim_{x \rightarrow 0} f(x) = \frac{\sin ax}{bx}$$

$$= \frac{a \cos ax}{b}$$

\therefore as $x \rightarrow 0$

$$\lim_{x \rightarrow 0} f(x) = \frac{a \cos a(0)}{b}$$

$$= \frac{a \times 1}{b} = \frac{a}{b}$$

hence $\lim_{x \rightarrow 0} f(x) = a/b$

$$\lim_{x \rightarrow 0} f(x) = a/b$$

(b) The model of a system has been developed to be as given in equation (1.2).

$$f(x) = 5x - 21$$

Given that $\delta = 0.1$ and $\Delta \delta = 0.01$, demonstrate, in tabular form, that the limit of the model as $x \rightarrow 6$ is equal to 9.

Solution

5 ± 0.1 68 ± 0.07 as $x \rightarrow 6$

$5.9 \leftarrow 6 \rightarrow 6.1$
 $L \leftarrow 6 \rightarrow L+E$

$L-E$	$q-h$	q	$q+h$	$L+E$
8.5	5.9	6	6.1	9.5
8.55	5.91		6.09	9.45
8.6	5.92		6.08	9.40
8.65	5.93		6.07	9.35
8.70	5.94		6.06	9.30
8.75	5.95		6.05	9.25
8.80	5.96		6.04	9.20
8.85	5.97		6.03	9.15
8.90	5.98		6.02	9.10
8.95	5.99		6.01	9.05
9.0	6.00		6.00	9.00

\therefore The limit of $f(x) = 5x - 21$ as $x \rightarrow 6$ is equal to 9.

c) Show whether the function given in equation (1.3) is continuous on the interval $(-5, 5)$. $f(x) = (25 - x^2)^{1/2}$

solution

For any function to be continuous right hand limit must be equal to the left hand limit. i.e. $f(a) = f(b)$

DH For RHL

$$\lim_{x \rightarrow 5} f(x) = (25 - x^2)^{1/2}$$

$$\lim_{h \rightarrow 0} f(x) = \lim_{h \rightarrow 0} (5+h)$$

$$\begin{aligned} &= (25 - (5+h)^2)^{1/2} \\ &= (25 - (5+0)^2)^{1/2} \end{aligned}$$

$$= 0$$

For LHL

$$\lim_{x \rightarrow 5^-} f(x) = (25 - x^2)^{1/2}$$

$$= \lim_{h \rightarrow 0} (5-h) = (25 - (5-0)^2)^{1/2}$$

$$= (25 - (5)^2)^{1/2}$$

$$= 0$$

Hence, $\lim_{x \rightarrow 5} f(x) = \lim_{x \rightarrow 5^-} f(x)$

i.e. $f(a) = f(b)$

The function $f(x) = (25 - x^2)^{1/2}$ is continuous on the interval $(-5, 5)$