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DEPT: MECHATRONICS ENGINEERING

MATRIC NO: 17ENGE051012

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Assuming  $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = k^2 + 5k + 6 = 0$$

$$k^2 + 2k + 3k + 6 = 0$$

$$k(k+3) + 3(k+2) = 0$$

$$(k+3)(k+2) = 0$$

$$k+3 = 0, \quad k+2 = 0$$

$$k_1 = -3 \quad \text{or} \quad k_2 = -2$$

$$C.F. = A e^{-3x} + B e^{-2x}$$

P.I.  $\Rightarrow F(x) = \cos t$

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

$$(-C \cos t - D \sin t) + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$5C \cos t - C \cos t + 6C \cos t - 5D \sin t - D \sin t + 6D \sin t = \cos t$$

$$5C + 5C = 1 \quad \text{--- (1)}$$

$$5D - 5D = 0 \quad \text{--- (2)}$$

Solving Simultaneously  $10C = 1$ ;  $C = \frac{1}{10}$

$$5\left(\frac{1}{10}\right) + 5C = 1 \quad \Rightarrow \frac{1}{2} + 5C = 1$$

$$5C = 1 - \frac{1}{2} = \frac{1}{2}$$

$$5C = \frac{1}{2}$$

$$C = \frac{1}{10}$$

$$P.I. = x = \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

Therefore  $G.S = P.I + C.F$

$$G.S = Ae^{-3x} + Be^{-2x} + \frac{1}{10} \sin x + \frac{1}{10} \cos x$$
$$= Ae^{-3x} + Be^{-2x} + \frac{1}{10} (\sin x + \cos x)$$

When  $t=0$ ,  $x=0.1$  &  $\frac{dx}{dt} = 0$

Substituting these values into equation

$$0.1 = A(1) + B(1) + \frac{1}{10}(1)$$

$$0.1 = A + B + 0.1$$

$$A + B = 0.1 - 0.1$$

$$A + B = 0 \quad \text{--- (1)}$$

$$\frac{dx}{dt} = -3Ae^{-3x} - 2Be^{-2x} - \frac{1}{10} \sin x + \frac{1}{10} \cos x$$

$$0 = -3A - 2B + \frac{1}{10}$$

$$3A + 2B = \frac{1}{10}$$

$$A + B = 0$$

$$3A + 2B = \frac{1}{10}$$

$$2A + 2B = 0$$

$$-(3A + 2B) = \frac{1}{10}$$

$$-A = 0.1 \approx \frac{1}{10}$$

$$A = 0.1$$

To find B we sub A into equ (1)

$$0.1 + B = 0$$

$$B = 0.1$$

$$G.S = 0.1e^{-3x} - 0.1e^{-2x} + \frac{1}{10} (\sin x + \cos x)$$
$$= 0.1 (e^{-3x} - e^{-2x} + \sin x + \cos x)$$

MATLAB

Command Window

Clear

clc

close all

Sym = t

t = 0:0.01:15

n = 0.1 \* [exp(-3\*t) - exp(-2\*t) + cos(t) + sin(t)]

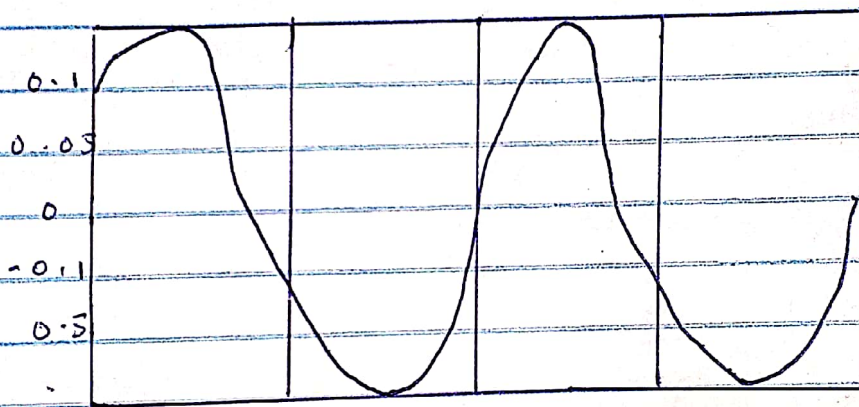
nn = sub(n)

Plot (t, nn)

axis tight

grid on

grid minor



$$n = k \sin(t, 10)$$

Knowing that  $n = 0.1$ , at  $t = 0$   $dn/dt = 0$

$$\frac{dn}{dt} = k \cos(t + \phi)$$

$$0 = k \cos(0 + \phi)$$

$$\therefore k \cos(\phi) = 0$$

$$0.1 = k \sin(0 + \phi)$$

$$k \sin(\phi) = 0.1 \quad \text{--- (1)}$$

$$\cos \phi = 0$$

$$\phi = \cos^{-1} 0$$

$$= 90^\circ$$

Substituting  $\phi$  into (1)

$$0.1 = k \sin(90)$$

$$k = \frac{0.1}{\sin(90)} = 0.1$$

$$\sin(90)$$

$$x = 0.1 [\sin(1+90)]$$

Command Window

Close all

Clear

Cle

Syms t, x

$$t = [0:0.01:15]$$

$$x = 0.1 (\sin(1+90))$$

plot (t,x)

