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17/ENGO7/019

Petroleum Engineering

ENR 381

Assignment 1

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Using Auxiliary Equation

Complementary Equation where $\cos t = 0$

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$$

$$m^2 + 5m + 6 = 0$$

$$m^2 + 2m + 3m + 6 = 0$$

$$(m^2 + 2m) + (3m + 6) = 0$$

$$m(m+2) + 3(m+2) = 0$$

$$(m+3)(m+2) = 0$$

$$m = -2 \text{ or } -3$$

$$x_{c.p} = Ae^{-2t} + Be^{-3t}$$

Particular Integral $x = \cos t = C \cos t + D \sin t$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

Substituting,

$$-C \cos t - D \sin t + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

Coefficient of $\cos t$

$$-C + 5D + 6C = 1$$

$$5C + 5D = 1 \quad \text{--- (1)}$$

Coefficient of $\sin t$

$$-D - 5C + 6D = 0$$

$$5D - 5C = 0 \quad \text{--- (2)}$$

$$5C + 5D = 1$$

$$-5C + 5D = 0$$

$$10D = 1$$

$$D = \frac{1}{10} = 0.1$$

from eqn 1

$$5C + 5D = 1$$

$$5C + 5(0.1) = 1$$

$$5C + 0.5 = 1$$

$$5C = 1 - 0.5$$

$$5C = 0.5$$

$$C = \frac{0.5}{5}$$

$$C = 0.1$$

General Soln

$$x = Ae^{-2t} + Be^{-3t} + 0.1\sin t + 0.1\cos t$$

$$\text{where } t=0, x=0.1, \frac{dx}{dt} = 0.$$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + 0.1 \times \sin(0) + 0.1 \cos(0)$$

$$0.1 = A + B + 0 + 0.1$$

$$0.1 - 0.1 = A + B$$

$$A + B = 0 \quad \text{--- 3)}$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + 0.1\cos t - 0.1\sin t$$

$$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} + 0.1\cos(0) - 0.1\sin(0)$$

$$0 = -2A - 3B + 0.1 + 0$$

$$-0.1 = -2A - 3B \quad \text{--- 4)}$$

$$\text{from eqn 3, } A = -B$$

$$-0.1 = -2(-B) - 3B$$

$$-0.1 = 2B - 3B$$

$$-0.1 = -B$$

$$B = 0.1$$

from eqn 3

$$A + 0.1 = 0$$

$$A = -0.1$$

$$x = -0.1e^{-2t} + 0.1e^{-3t} + 0.1\cos t + 0.1\sin t$$

2. Command window

clc

clear

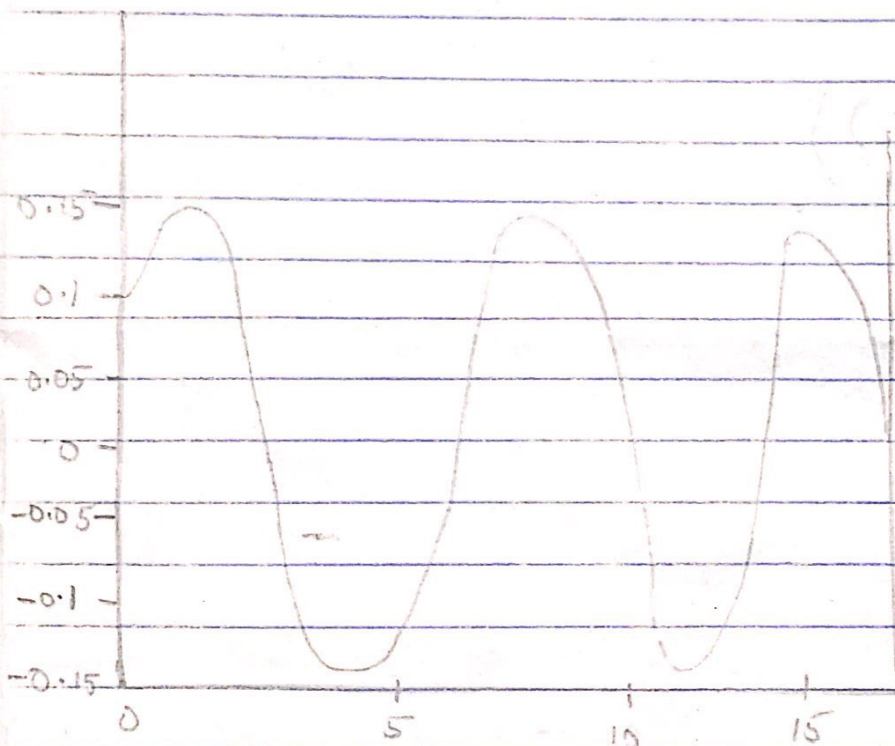
close all

Syms t

t = 0:0.01:15

x = -0.1 * exp(-2*t) + 0.1 * exp(-3*t) + 0.1 * cos(t) + 0.1 * sin(t)

plot(t, x)



3. Steady state form.

$$x_{t \rightarrow \infty} = x_{s-s} = 0.1\cos t + 0.1\sin t$$

$$0.1\cos t + 0.1\sin t = K\sin(t+a)$$

$$K\sin(t+a) = K\sin t \cos a + K\cos t \sin a$$

Coefficient of $\cos t$

$$0.1 = K\sin a$$

Coefficient of $\sin t$

$$0.1 = K\cos a$$

Squaring both sides

$$K^2 \sin^2 a + K^2 \cos^2 a = 0.01 + 0.01$$

$$K^2 (\sin^2 a + \cos^2 a) = 0.02, \text{ where } (\sin^2 a + \cos^2 a) = 1$$

$$K^2 = 0.02$$

$$K = \sqrt{0.02}$$

$$K = 0.1414 = \frac{\sqrt{2}}{10}$$

$$\frac{K \sin a}{K \cos a} = \frac{0.1}{0.1} = 1$$

$$\tan a = 1$$

$$\tan^{-1}(1) = a$$

$$a = 45^\circ = \frac{\pi}{4}$$

Steady state.

$$x_{s-s} = \frac{\sqrt{2}}{10} \left(\sin \left(t + \frac{\pi}{4} \right) \right)$$