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The dynamic model of a body in motion performing damped forced vibrations is as in Equation (1)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that when $t=0$, $x=0.1$ and $dx/dt = 0$

(a) Using the Auxiliary Equation method, obtain the solution of the model in form of an expression having x as a function of t.

(b) with the aid of a MATLAB mfile program, plot the relationship between x and t for $0 \le t \le 15$ time unit using a step size of 0.01 unit and

(c) write the steady-state solution of the model in form of $x = k\sin(t+a)$

(a) $\frac{d^2x}{dt^2} + 5dx + 6x = \cos t$ ... eqn (1)

C.F; $m^2+5m+6=0$

$m^2+3m+2m+6=0$

$m(m+3)+2(m+3)=0$

$(m+2)(m+3)=0$

$m=-2$ or $m=-3$

C.F: $x = Ae^{-2t} + Be^{-3t}$

P.I:

$x = C\cos t + D\sin t$

$dx/dt = -C\sin t + D\cos t$

$d^2y/dt^2 = -C\cos t - D\sin t$

Substituting x, dx/dt and d^2y/dt^2 in eqn (1),

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$-C\cos t + 5D\cos t + 6C\cos t - D\sin t - 5C\sin t + 6D\sin t = \cos t$

$\cos t(-C + 5D + 6C) + \sin t(-D - 5C + 6D) = \cos t$

$\cos t(5D + 5C) + \sin t(5D - 5C) = \cos t$

Equating coefficients:

$5D + 5C = 1$ ... eqn (3)

$5D - 5C = 0$ ... eqn (4)

Adding eqn (3) and eqn (4)

$10D = 1 \therefore D = \frac{1}{10}$

Substituting the value of D in eqn (3)

$5\left(\frac{1}{10}\right) + 5C = 1$

$\frac{5}{10} + 5C = 1$

$0.5 + 5C = 1$

$5C = 1 - 0.5 = 0.5$

$C = 0.5/5 = 1/10$

P.I.: $x = \frac{1}{10}\cos t + \frac{1}{10}\sin t = \frac{1}{10}(\cos t + \sin t)$

The general solution is $x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}(\cos t + \sin t)$ ... eqn (5)

When $t = 0$, eqn (5) becomes

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}(\cos(0) + \sin(0))$

$0.1 = A + B + 1/10$

$1/10 = A + B + 1/10$

$A + B = 1/10 - 1/10 = 0 \therefore A + B = 0$ ... eqn (6)

When $dx/dt = 0$;

$dx/dt = \frac{d\left(Ae^{-2t} + Be^{-3t} + 1/10(\cos t + \sin t)\right)}{dt}$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$$

$$-2Ae^{-2t} - 3Be^{-3t} - \tfrac{1}{10}\sin t + \tfrac{1}{10}\cos t = 0$$

Substituting $t = 0$;

$$-2Ae^{-2(0)} - 3Be^{-3(0)} - \frac{1}{10}\sin(0) + \frac{1}{10}\cos(0) = 0$$

$$-2A - 3B + \tfrac{1}{10} = 0$$

$$-(2A+3B) = -0.1$$

$$2A+3B = 0.1 \quad \ldots \text{eqn}(7)$$

Multiplying eqn (6) by 3,

$$3(A+B=0) = 3A+3B=0 \quad \ldots \text{eqn}(8)$$

Subtracting eqn (7) from eqn (8)

$$2A+3B = 0.1$$

$$3A+3B = 0$$

$$A = 0 - 0.1 = -0.1$$

From eqn (6), $-0.1 + B = 0 \ \therefore\ B = 0.1$

Substituting the values back in the general solution,

$$x = -0.1e^{-2t} + 0.1e^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$$

$$x = \frac{-1}{10}e^{-2t} + \frac{1}{10}e^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$$

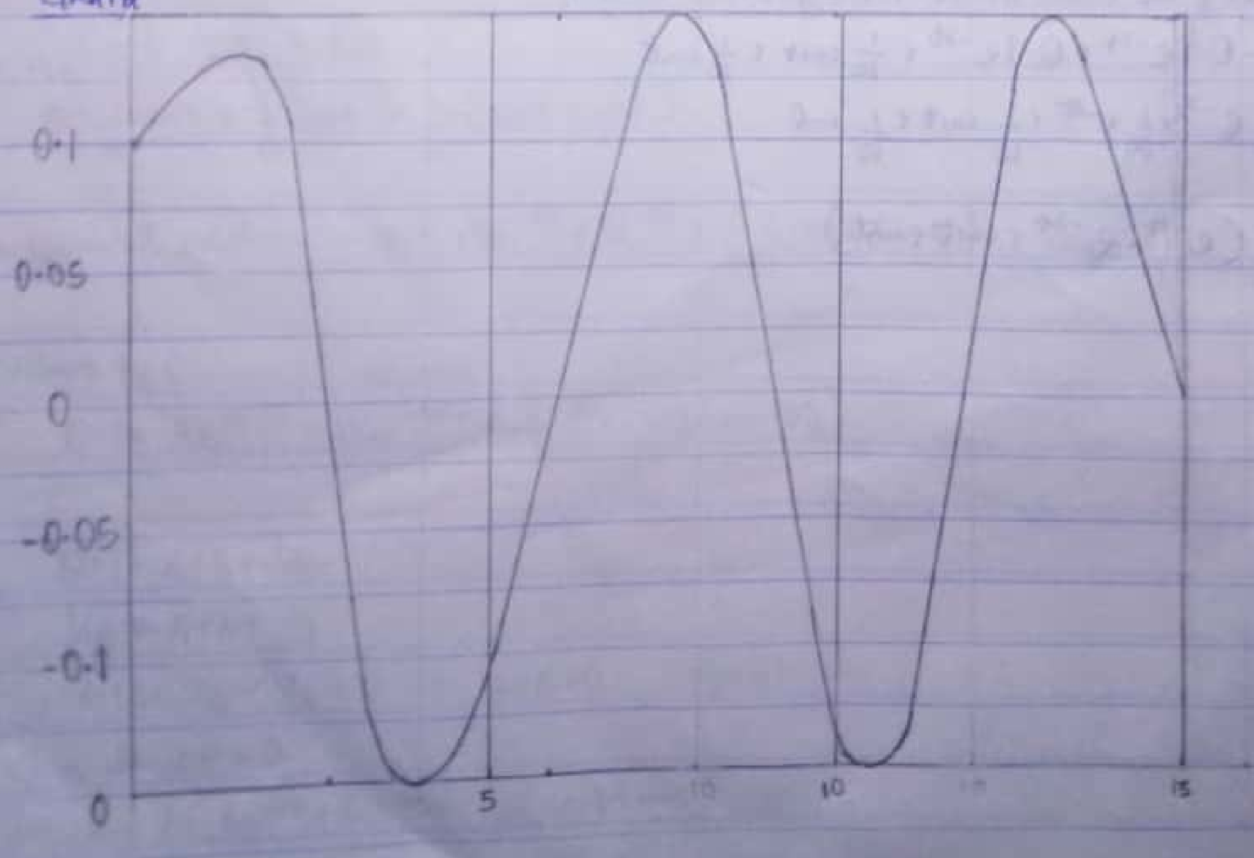
$$x = \frac{1}{10}\left(e^{-3t} - e^{-2t} + \cos t + \sin t\right)$$

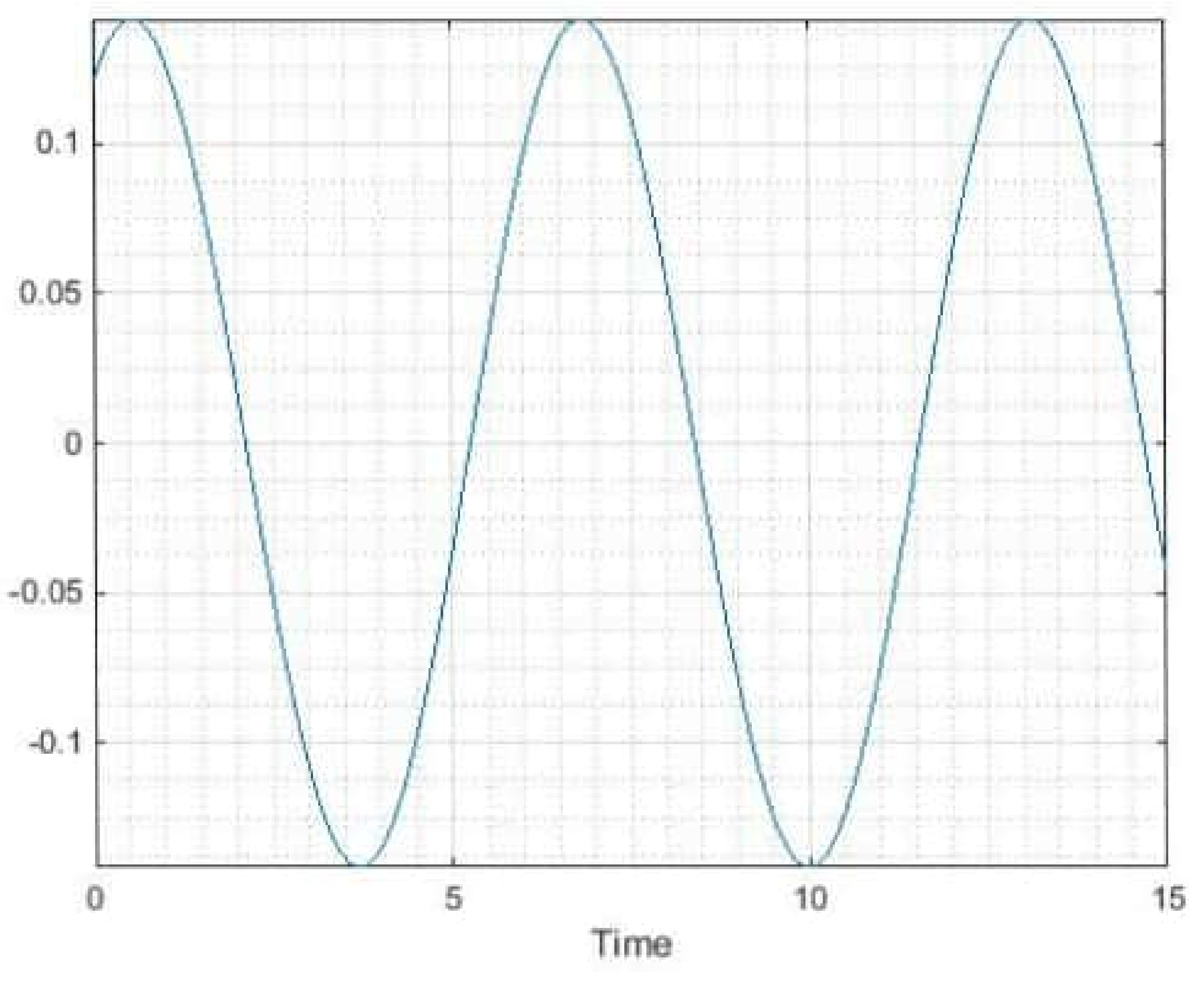
① MATLAB mfile program

commandwindow

```
clear
clc
close all
t = 0: 0.01 : 15
x = 0.1 *(exp(-3*t) - exp(-2*t) + cos(t) + sin(t))
x1 = subs (x)
plot(t, x1)
xlabel ('t')
ylabel ('x')
grid on
grid minor
axis tight
```

GRAPH




0.1
0.05
0
-0.05
-0.1
0
5
10
15
Time

(c) $x_{ss} = x_{t\to\infty} = 0.1\cos t + 0.1\sin t = k\sin(t+a)$

$x_{ss} = k\sin t\cos a + k\cos t\sin a$

$0.1 = k\sin a \; ; \; 0.1 = k\cos a$

$k^2\sin^2 a + k^2\cos^2 a = \frac{1}{100} + \frac{1}{100}$

$k^2(\sin^2 a + \cos^2 a) = \frac{2}{100}$

$k^2 = \frac{2}{100}$ (since $\sin^2 a + \cos^2 a = 1$)

$k = \frac{\sqrt{2}}{10}$

$\therefore \frac{k\sin a}{k\cos a} = \frac{0.1}{0.1} = 1$

$\tan a = 1, a = \pi/4$

$x_{ss} = \frac{\sqrt{2}}{10}\sin\left(t + \pi/4\right)$

The steady-state solution of the model is $\frac{\sqrt{2}}{10}\sin(t+\pi/4)$.

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