28-09-019

Ogiora Peace Doutimi
Civil Engineering
17/Eng03/1048

Assignment I

Question 1

The dynamic model of a body in motion performing damped forced vibrations is as in equation (1)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that when $t = 0$, $x = 0.1$ and $\frac{dx}{dt} = 0$;

a) Using the auxiliary equation method, obtain the solution of the model in form of an expression having x as a function of t;

Solution.

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$m^2 + 5m + 6 = 0 \quad (\text{i.e homogeneity}).$$

$$m^2 + 2m + 3m + 6 = 0$$

$$(m^2 + 2m) + (3m + 6) = 0$$

$$m(m+2) + 3(m+2) = 0$$

$$(m+3)(m+2) = 0$$

$$m = -2 \;/\; -3$$

$$x_{cf} = Ae^{-2t} + Be^{-3t}$$

Particular Integral $x = \cos t$

$$= C\cos t + D\sin t$$

$$\frac{dx}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$$

Substitution

$$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$$

$$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$$

Coefficient of $\cos t$

$$-C + 5D + 6C = 1$$

$$5C + 5D = 1 \quad \text{---} (i)$$

Coefficient of $\sin t$

$$-D - 5C + 6D = 0$$

$$5D - 5C = 0 \quad \text{---} (ii)$$

$5C + 5D = 1$

$-5C + 5D = 0$

$10D = 1$

$D = \frac{1}{10} = 0.1$

from eqn 1

$5C + 5D = 1$

$5C + 5(0.1) = 1$

$5C + 0.5 = 1$

$5C = 0.5$

$C = \frac{0.5}{5}$ $\therefore C = 0.1$

General Sol

$x_c = AP^{-2t} + BP^{-3t} + 0.1\sin t + 0.1\cos t$

where $t = 0$, $x = 0.1$

$\frac{dt}{dx} = 0$

$0.1 = AP^{-2(0)} + BP^{-3(0)} + 0.1 \times \sin(0) + 0.1\cos(0)$

$0.1 = A + B + 0 + 0.1$

$0.1 = 0.1 = A + B$

$\therefore A + B = 0$ ——— (iii)

$\frac{dx_c}{dt} = -2AP^{-2t} - 3BP^{-3t} + 0.1\cos t - 0.1\sin t$

$0 = -2AP^{-2(0)} - 3BP^{-3(0)} + 0.1\cos(0) - 0.1\sin(0)$

$0 = -2A - 3B + 0.1 + 0$

$-0.1 = -2A - 3B$ ——— (iv)

from eq (iii), $A = -B$

$-0.1 = -2(-B) - 3B$

$-0.1 = 2B - 3B$

$-0.1 = -B$

$B = 0.1$

from eq (iii)

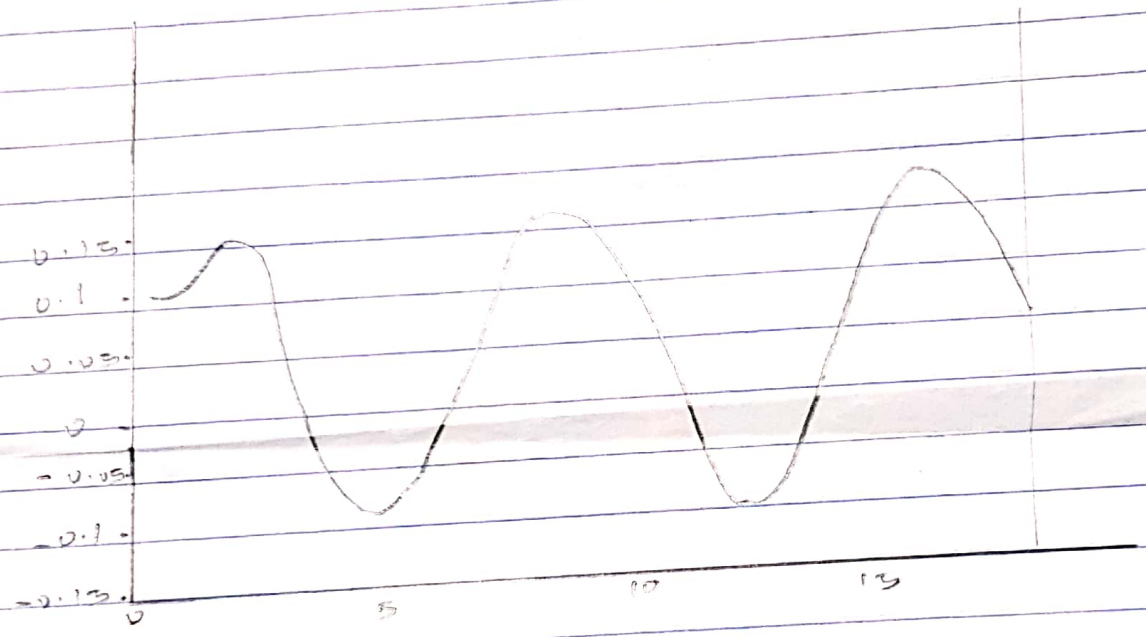
$A + 0.1 = 0$

$A = -0.1$

$x = -0.1P^{-2t} + 0.1P^{-3t} + 0.1\cos t + 0.1\sin t$

2) Command window

```
clc
clear
close all
syms t
t = 0:0.01:15
x = -0.1*exp(-2*t) + 0.1*exp(-3*t) + 0.1*cos(t) + 0.1*sin(t)
plot(t, x).
```



3) steady state

$x \to x = 0.1\cos t + 0.1\sin t$

$t \to \infty$

$0.1\cos t + 0.1\sin t = k\sin(t + a)$

$k\sin(t+a) = k\sin t\cos a + k\cos t\sin a$

Coefficient of cost

$0.1 = k\sin a$

Coefficient of sint

$0.1 = k\cos a$

$k^2\sin^2 a + k^2\cos^2 a = 0.01 + 0.01$

$k^2(\sin^2 a + \cos^2 a) = 0.02$

$k(\sin^2 a + \cos^2 a) = 1$

$k^2 = 0.02$

$k = \sqrt{0.02}$

$k = 0.1414 = 52/10.$

$$\tan a = 1$$

$$\tan^{-1}(1) = a$$

$$a = 45^\circ = \frac{\pi}{4}$$

Steady state

$$x_{ss} = \frac{\sqrt{2}}{10}\left(\sin\left(t + \frac{\pi}{4}\right)\right)$$