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Assignment 1

1 The dynamic model of a body in motion performing damped force vibration is an equation (1)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that when $t = 0$, $x = 0.1$ and $\frac{dx}{dt} = 0$

a) Using the auxillary equation, Obtain the solution of the model in form of an expression having x as a function of t

b) With aid of MATLAB mfile program put the relationship between x and t for $0 \leq t \leq 13$ time unit using a step size of 0.01 unit and

c) Write the steady State solution of the model in form of $x = K\sin(t+a)$

Solution

a) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

Auxillary Equation: $m^2 + 5m + 6 = 0$

$m^2 + 2m + 3m + 6 = 0$

$m(m+2) + 3(m+2) = 0$

$(m+3)(m+2) = 0$

$m = -3$ or -2

Complementary function: $X = Ae^{-3t} + Be^{-2t}$

Particular integral: Assume $x = C\cos t + D\sin t$

$$\frac{dx}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$$

Putting the value of d^2x/dt^2 & $\frac{dx}{dt}$ in the equation

Derivative

$$-(C\cos t - D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$$

$$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$$

$$-C\cos t + 5D\cos t + 6C\cos t - D\sin t - 5C\sin t + 6D\sin t = \cos t$$

Coefficient of $\cos t$: $-C + 5D + 6C = 1$ ... (1)

Coefficient of $\sin t$: $-D - 5C + 6D = 0$ ... (2)

from equation (1): $5C + 5D = 1$ ... (3)

from equation (2): $5D - 5C = 0$ ... (4)

from equation (3): $5D = 1 - 5C$

Put equation (3) in 4

$$1 - 5C - 5C = 0$$

$$1 - 10C = 0$$

$$10C = 1, \quad C = \frac{1}{10} \quad \text{... (5)}$$

put equation 5 in 4

$$5D - \frac{1}{2} = 0$$

$D = 0.1$

~~Particular~~ So general solution: $x = Ae^{-3t} + Be^{-3t} + 0.1 + 0.1\cos t + 0.1\sin t$

Particular Solution when $t = 0$ & $x = 0.1$, $\frac{dx}{dt} = 0$

$0.1 = A + B + 0.1$

$A + B = 0$ ... (1)

$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} - 0.1\sin t + 0.1\cos t$

$0 = -3A - 2B + 0.1$ ... (2)

from equation (1)

$B = -A$ ... (3)

Put equation 3 in (2)

$0 = -3A - 2(-A) + 0.1$

$0 = -3A + 2A + 0.1$

$0 = -A + 0.1$

$A = 0.1$

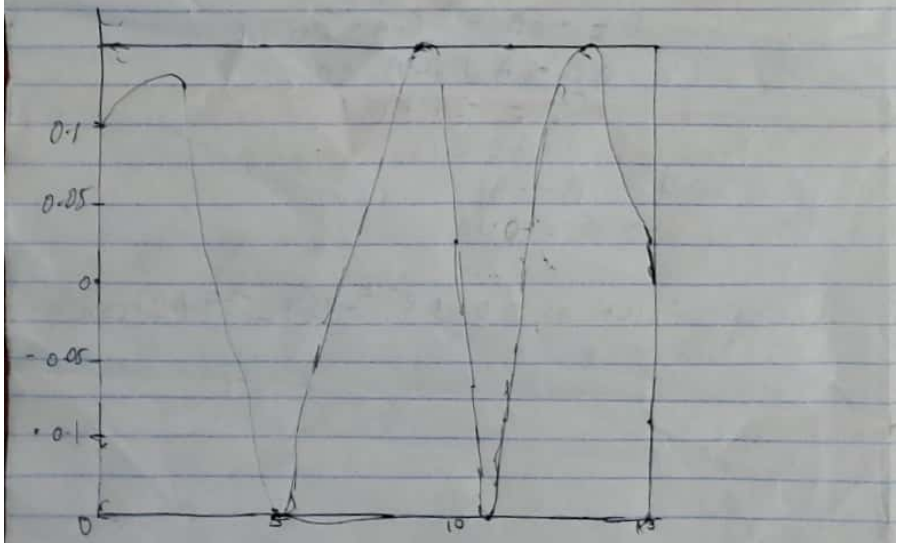
and $B = -A$

so $B = -0.1$

General Solution: $0.1e^{-3t} - 0.1e^{-2t} + 0.1\cos t + 0.1\sin t$

b) Command window

```
Clear
clc
Close all
Syms x t
x = 0.1 * exp(-3 * t) - 0.1 * exp(-2 * t) + 0.1 * cos(t) +
0.1 * sin(t)
t = 0: 0.01: 15
xn = subs(x)
Plot(t, xn)
xlabel('time')
grid on
grid major
axis tight
```



(c) $0.1\cos t + 0.1\sin t = K\sin(t+a)$ at steady flow

$0.1\cos t + 0.1\sin t = K\sin t\cos a + K\sin a\cos t$

Comparing coefficient

Comparing of coefficient

Coefficient of cost $= 0.1 = K\sin a$

Coefficient of sint: $0.1 = K\cos a$

Square $K\sin a$ and $K\cos a$ and equate it to the addition

$K^2\sin^2 a + K^2\cos^2 a = 0.01$

$K^2(\sin^2 a + \cos^2 a) = 0.2$

$K^2 = 0.2$

$K^2 = \frac{2}{10}$

$K = \sqrt{\frac{2}{10}}$

$K\sin a = \frac{0.1}{K\cos a} \quad \frac{0.1}{0.1}$

from tana: 1

at $a = \tan^{-1}(1)$

$a = 45^\circ$ or $\frac{\pi}{4}$

$\therefore K$ steady state; $K_{ss} = \sqrt{\frac{2}{10}}\sin\left(\frac{\pi}{4} + t\right)$