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Course: ENG 381

Assignment 1

① The dynamic model of a body in motion performing damped force vibration is an equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that when $t=0$, $x = 0.1$ and $\frac{dx}{dt} = 0$

ⓐ Using the auxillary equation, obtain the solution of the model in form of an expression having x as a function of t

ⓑ with aid of MATLAB mfile program put the relationship between x and t for $0 \leq 1 \leq 13$ time unit using a step size of 0.01 unit and

ⓒ write the steady state solution of the model in form of $x = K$ sin

Solution

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Auxillary Equation: $m^2 + 5m + 6 = 0$

$$m^2 + 2m + 3m + 6 = 0$$

$$m(m+2) + 3(m+2) = 0$$

$$(m+3)(m+2) = 0$$

$$m = -3 \text{ or } -2$$

Complementary function: $x = Ae^{-3t} + Be^{-2t}$

particular integral: Assume $x = C\cos t + D\sin t$

$$\therefore \frac{dx}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$$

Putting the value of d^2x/dt^2 & $\frac{dx}{dt}$ in the equation

$-(C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(-C\cos t + D\sin t)$

$= \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t +$

$6D\sin t = \cos t$

$-C\cos t + 5D\cos t + 6\cos t - D\sin t - 5C\sin t +$

$6D\sin t = \cos t$

$-C\cos t + 5D\cos t + 6\cos t - D\sin t - 5C\sin t +$

$6D\sin t : \cos t$

Coefficient of cost: $-C + 5D + 6C = 1$ --- ①

Coefficient of sint: $-D - 5C + 6D = 0$ --- ②

From equation ① $= 5C + 5D = 1$ --- ③

From equation ② $= 5D - 5C = 0$ --- ④

From equation ③ $= 5D = 1 - 5C$

put equation ③ in 4

$= 1 - 5C - 5C = 0$

$1 - 10C = 0$

$10C = 1, C = 1/10$ --- ⑤

put equation 5 in 4

$+5D - \frac{1}{2} = 0$

$D = 0.1$

general Solution: $x = Ae^{-3t} + Be^{-3t} + 0.1\cos t +$

$0.1 \sin t.$

Particular solution when $t = 0, x = 0.1, \frac{dx}{dt} = 0$

$0.1 = A + B + 0.1$

$A + B = 0$

$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} - 0.1\sin t + 0.1\cos t$

$0 = -3A - 2B + 0.1$ --- ②

From Equation ... ①

$B = -A$ ... ⑥

put equation ⑥ in ②

$0 = -3A - 2(-A) + 0.1$

$0 = -3A + 2A + 0.1$

$0 = -A + 0.1$

$A = 0.1$

and $B = -A$

so $B = -0.1$

General solution: $0.1e^{-3t} - 0.1e^{-2t} + 0.1\cos t + 0.1\sin t$.

(c) $0.1\cos t + 0.1\sin t = K\sin(t+a)$ at steady flow

$0.1\cos t + 0.1\sin t = K\sin t\cos a + K\sin a\cos t$

Comparing coefficient

Comparing of coefficient

Coefficient of $\cos t$ = $0.1 = K\sin a$

Coefficient of $\sin t$: $0.1 = K\cos a$

Square $K\sin a$ and $K\cos a$ and equate it to the addition

$$K^2\sin^2 a + K^2\cos^2 a = 0.1 + 0.1$$

$$K^2(\sin^2 a + \cos^2 a) = 0.2$$

$$K^2 = 0.2$$

$$K^2 = \frac{2}{10}$$

$$K = \sqrt{\frac{2}{10}}$$

$$\frac{K\sin a}{K\cos a} = \frac{0.1}{0.1}$$

from $\tan a = 1$

$$a = \tan^{-1}(1)$$

$$a = 45^\circ \text{ or } \frac{\pi}{4}$$

$\therefore$ K steady state; $K_{ss} = \sqrt{\frac{2}{10}}\sin\left(\frac{\pi}{4} + t\right)$