

$$1. \frac{d^2 x}{dt^2} + 5 \frac{dx}{dt} + 6x = \cos t$$

$$A.E: k^2 + 5k + 6 = 0$$

$$k^2 + 2k + 3k + 6 = 0$$

$$k(k+2) + 3(k+2) = 0$$

$$k_1 = -3, k_2 = -2$$

$$x_{CF} = Ae^{-3x} + Be^{-2x}$$

P.I

$$F(x) = \cos t$$

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2 x}{dt^2} = -C \cos t - D \sin t$$

$$[-C \cos t - D \sin t] + -5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$\cos t: -C + 5D + 6C = 1 \quad \dots \quad (1)$$

$$\sin t: -D - 5C + 6D = 0 \quad \dots \quad (2)$$

$$\sin t: -D - 5C + 6D = 0$$

$$-5C + 5D = 0 \quad \dots \quad (2)$$

$$\text{Eqn (1) + (2): } 10D = 1$$

$$D = \frac{1}{10}$$

Subst. D into (1)

$$5C + 5\left(\frac{1}{10}\right) = 1$$

$$5C = 1 - \frac{1}{2}$$

$$C = \frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$$

$$x_{PI} = \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$x_{G.S} = Ae^{-3t} + Be^{-2t} + \frac{1}{10} (\cos t + \sin t)$$

When  $t = 0$ ,  $x = 0.1$  and  $\frac{dx}{dt} = 0$

$$0.1 = A(1) + B(1) + \frac{1}{10}(1)$$

$$0.1 = A + B + 0.1$$

$$A + B = 0.1 - 0.1$$

$$A + B = 0 \quad \dots \dots \dots (1)$$

$$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$$

$$0 = -3A - 2B + \frac{1}{10}$$

$$3A + 2B = \frac{1}{10} \quad \dots \dots \dots (2)$$

$$A + B = 0 \quad \dots \dots \times 2$$

$$3A + 2B = 0.1 \quad \dots \dots \times 1$$

$$2A + 2B = 0$$

$$3A + 2B = 0.1$$

$$-A = -0.1$$

$$A = 0.1$$

To find B

$$0.1 + B = 0$$

$$B = -\frac{1}{10} \text{ or } -0.1$$

$$x(t) = \frac{1}{10}e^{-3t} - \frac{1}{10}e^{-2t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$$

$$= \frac{1}{10} (e^{-3t} - e^{-2t} + \cos t + \sin t)$$

iii) MATLAB mFile

Command window

Clear

clc

close all

syms t

t = 0:0.01:15

x = 0.1 \* [exp(-3\*t) - exp(-2\*t) + cos(t) + sin(t)]

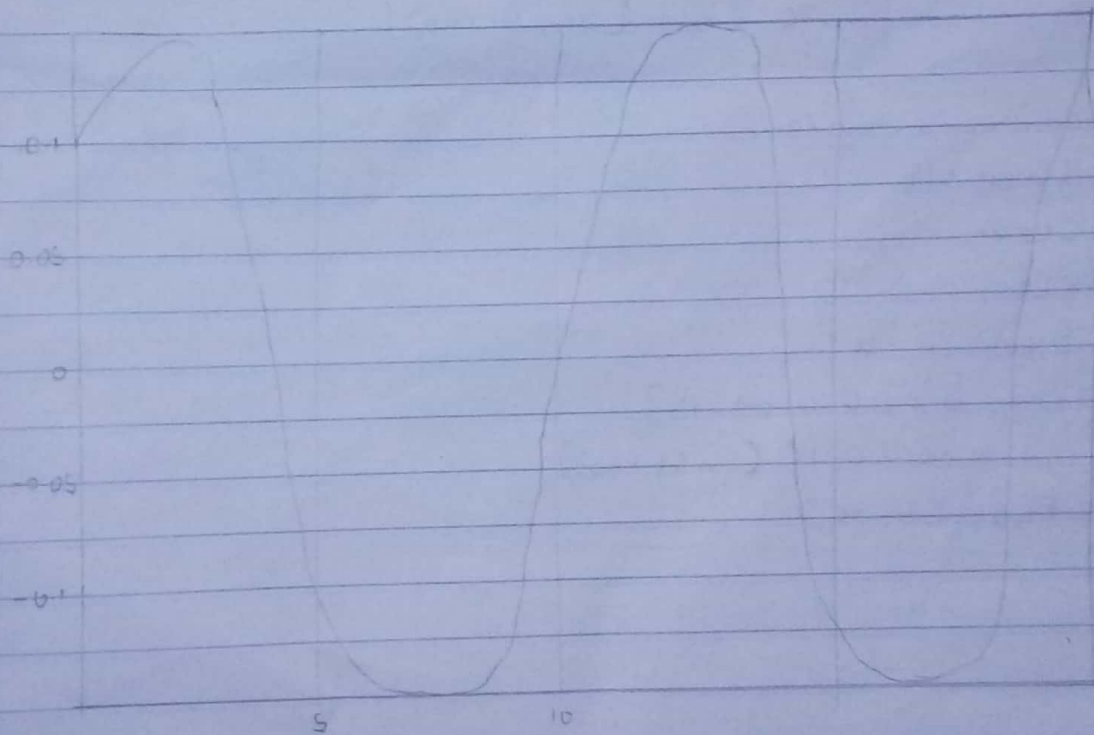
xn = subs(x)

plot(t, xn)

axis tight

grid on

grid minor





$$x = k \sin(t + a)$$

Knowing  $x = 0.1$  at  $t = 0$  &  $\frac{dx}{dt} = 0$

$$\frac{dx}{dt} = k \cos(t + a)$$

$$0 = k \cos(0 + a)$$

$$k \cos(a) = 0 \quad \dots \dots \dots$$

$$0.1 = k \sin(0 + a)$$

$$k \sin(a) = 0.1 \quad \dots \dots \dots (1)$$

$$\cos(a) = 0$$

$$a = \cos^{-1} 0$$

$$= 90^\circ$$

Sub  $a$  into (1)

$$0.1 = k \sin(90)$$

$$k = \frac{0.1}{\sin 90} = k = 0.1$$

$$x = 0.1 [\sin(t + 90)]$$

Command window

Close all

Clear

clc

Syms t, x

$$t = [0:0.01:15]$$

$$x = 0.1 * [\sin(t + 90)]$$

Plot(t, x)