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Civil Engineering
17/ENG03/015



ENG 381

Assignment 1

$$(9) \frac{d^2x}{dt^2} + 5 \frac{dx}{dt} + 6x = \cos t$$

$$\text{Auxilliary equation } \Rightarrow k^2 + 5k + 6 = 0$$

$$k^2 + 2k + 3k + 6 = 0$$

$$k(k+2) + 3(k+2) = 0$$

$$(k+3)(k+2) = 0$$

$$\therefore k+3=0 \quad \text{or} \quad k+2=0$$

$$k = -3$$

$$k = -2$$

$$\text{Complementary function } \Rightarrow y = Ae^{k_1 x} + Be^{k_2 x}$$

$$y = Ae^{-3x} + Be^{-2x}$$

$$\text{P.I } x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

$$\Rightarrow -C \cos t - D \sin t + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$\Rightarrow -C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$\Rightarrow C \cos t + 6C \cos t - D \sin t + 6D \sin t - 5C \sin t + 5D \cos t = \cos t$$

$$\Rightarrow 5C \cos t + 5D \sin t - 5C \sin t + 5D \cos t = \cos t$$

Comparing coefficient

$$\text{Coefficient of } \cos: -C + 5D + 6C = 1 \quad \text{--- (1)}$$

$$\text{Coefficient of } \sin: -D - 5C + 6D = 0 \quad \text{--- (2)}$$

from (1)

$$5C + 5D = 1 \quad \text{--- (3)}$$

$$\text{from (2) } 5D - 5C = 0$$

$$5D - 5C = 0 \quad \text{--- (4)}$$

$$\text{from (1) } 5D = 1 - 5C \quad \text{--- (5)}$$

Substitute ⑤ into ④

$$(1-5C)-5C=0$$

$$1-5C-5C=0$$

$$1-10C=0$$

$$C = \frac{1}{10}$$

Substitute $C = \frac{1}{10}$ into ⑤

$$5D = 1 - 5C$$

$$5D = 1 - 5 \frac{1}{10}$$

$$5D = \frac{1}{2}$$

$$D = \frac{1}{10}$$

$$y_{p.I} = \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$y = y_x + y_{p.I} \\ = Ae^{-3x} + Be^{-2x} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

When $t=0$, $x=0.1$ and $\frac{dx}{dt} = 0$

$$0.1 = A + B + 0.1$$

$$A + B = 0 \quad \text{--- (6)}$$

$$\frac{dy}{dx} = -3Ae^{-3x} - 2Be^{-2x} + \frac{1}{10} \sin t + \frac{1}{10} \cos t$$

$$0 = -3A - 2B + \frac{1}{10}$$

from ⑥ $B = -A$

$$0 = -3A - 2(-A) + 0.1$$

$$0 = -A + 0.1$$

$$A = 0.1$$

when

$$B = -A$$

$$B = -0.1$$

General solution

$$0.1e^{-3t} - 0.1e^{-2t} + 0.1\sin t + 0.1\cos t$$

$$(c) 0.1\cos t + 0.1\sin t = K\sin(\pi/4) \text{ at steady flow}$$

$$0.1\cos t + 0.1\sin t = K\sin t \cos \alpha + K\sin \alpha \cos t$$

Comparing coefficient

$$\text{Coefficient of } \cos t: 0.1 = K\sin \alpha$$

$$\text{Coefficient of } \sin t: 0.1 = K\cos \alpha$$

Square $K\sin \alpha$ and $K\cos \alpha$ and equate it to the addition

$$K^2\sin^2 \alpha + K^2\cos^2 \alpha = 0.1 + 0.1$$

$$K^2(\sin^2 \alpha + \cos^2 \alpha) = 0.2$$

$$K^2 = 0.2$$

$$K^2 = \frac{2}{10} \quad (\sin^2 \alpha + \cos^2 \alpha = 1)$$

$$K = \frac{\sqrt{2}}{10}$$

$$K = \frac{\sqrt{2}}{10}$$

$$\frac{K\sin \alpha}{K\sin \alpha} = \frac{0.1}{0.1}$$

$$\frac{K\cos \alpha}{K\cos \alpha} = \frac{0.1}{0.1}$$

from $\alpha = 1$

$$\alpha = \tan^{-1}(1)$$

$$\alpha = 45^\circ \text{ or } \frac{\pi}{4}$$

$$\therefore K \text{ steady state; } K_{ss} = \frac{\sqrt{2}}{10} \sin\left(\frac{\pi}{4} + t\right)$$

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(b) 1 Commandwindow
    2 Clear
    3clc
    4 close all
    5 syms t
    6 x = (1/10*EXP(-3*t)) - (1/10*EXP(-2*t)) + (1/10*(cos(t) + sin(t)))
    7 t = 0:0.01:15
    8 xt = subs(x,t)
    9 xtn = double(xt)
    10 Plot(t,xtn)
    11 xlabel('t')
    12 ylabel('x')
    13 grid on
    14 grid minor
    15 axis tight

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