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17/ENG04/051

ELECT/ELECT
ENG 381

1) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

Assuming $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$

$\therefore \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x \equiv k^2 + 5k + 6 = 0$

$k^2 + 2k + 3k + 6 = 0$

$k(k+2) + 3(k+2) = 0$

$(k+3)(k+2) = 0$

$k+3 = 0 \quad , \quad k+2 = 0$

$\therefore K_1 = -3$ or $K_2 = -2$

$\therefore$ CF $\Rightarrow Ae^{-3x} + Be^{-2x}$

P.I $\Rightarrow F(x) = \cos t$

$\therefore x = C\cos t + D\sin t$

$\therefore \frac{dx}{dt} = -C\sin t + D\cos t$

$\therefore \frac{d^2x}{dt^2} = -C\cos t - D\sin t$

$\therefore (-C\cos t - D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$\therefore 5D\cos t - C\cos t + 6C\cos t - 5C\sin t - D\sin t + 6D\sin t = \cos t$

$5D + 5C = 1$ --- (1)

$5D - 5C = 0$ --- (2)

Solving simultaneously $\quad 10D = 1 \quad \therefore D = 1/10$

substituting $D = 1/10$ into (1)

$\Rightarrow 5(1/10) + 5C = 1 \quad \Rightarrow \quad \frac{1}{2} + 5C = 1$

$\Rightarrow 5C = 1 - 1/2 \quad \Rightarrow \quad 5C = 1/2$

$C = 1/10$

$\therefore$ P.I $\Rightarrow x = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

Therefore GS => P.I + C.F

$$\Rightarrow Ae^{-3x} + Be^{-2x} + \frac{1}{10}\sin t + \frac{1}{10}\cos t.$$

$$\Rightarrow Ae^{-3x} + Be^{-2x} + \frac{1}{10}(\sin t + \cos t).$$

when $t = 0$, $x = 0.1$ and $\frac{dx}{dt} = 0$

Substituting these values into the equation

$$0.1 = A(1) + B(1) + \frac{1}{10}(1)$$

$$0.1 = A + B + 0.1$$

$$A + B = 0.1 - 0.1$$

$$A + B = 0 \quad \text{-- (1)}$$

$$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$$

$$0 = -3A - 2B + \frac{1}{10}$$

$$3A + 2B = \frac{1}{10} \quad \text{---- (2)}$$

$$A + B = 0 \quad \cdots \times 2$$

$$3A + 2B = 1/10 \quad \cdots \times 1$$

$$2A + 2B = 0$$

$$-(3A + 2B) = \frac{1}{10}$$

$$-A = -0.1$$

$$\therefore A = 0.1$$

To find B we substitute A into equ (1)

$$0.1 + B = 0$$

$$\therefore B = -0.1$$

$$\therefore G.S = 0.1e^{-3t} - 0.1e^{-2t} + 1/10(\sin t + \cos t)$$

$$= 0.1(e^{-3t} - e^{-2t} + \sin t + \cos t).$$

MATLAB

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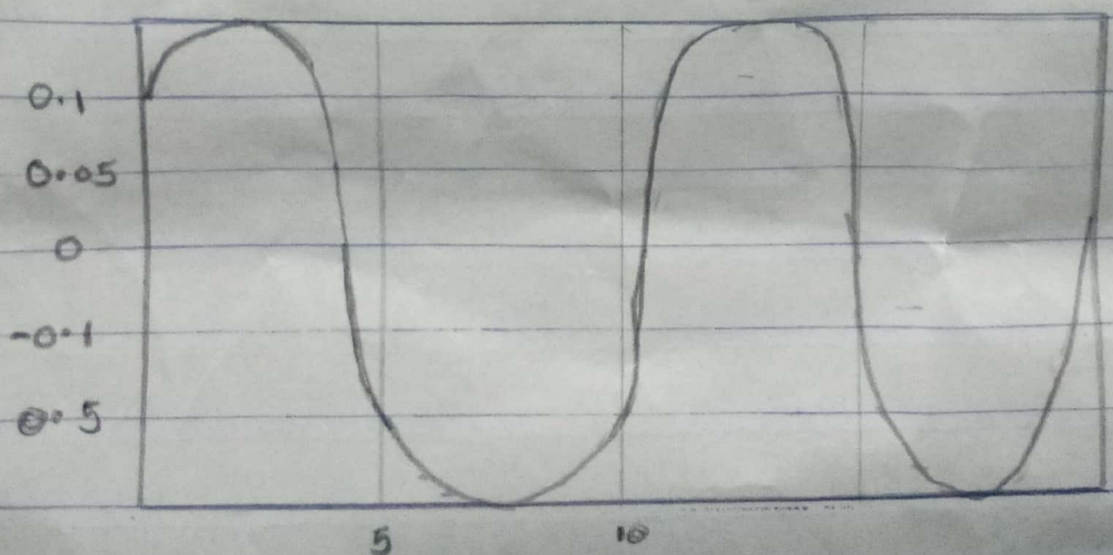
```
clear
Clc
Close all
Syms t
t = 0: 0.01: 15
x = 0.1* [exp(-3*t) - exp(-2*t) + Cos(t) + Sin(t)]
xn = Subs (x)
plot (t, xn)
axis tight
grid on
grid minor
```



(c) $x_{ss} = x_{t \to \infty} = 0.1 \cos t + 0.1 \sin t = k \sin(t + a)$

$x_{ss} = k \sin t \cos a + k \cos t \sin a$

$0.1 = k \sin a \; ; \; 0.1 = k \cos a$

$$k^2 \sin^2 a + k^2 \cos^2 a = \frac{1}{100} + \frac{1}{100}$$

$$k^2 (\sin^2 a + \cos^2 a) = \frac{2}{100}$$

$$k^2 = \frac{2}{100} \quad (\text{since } \sin^2 a + \cos^2 a = 1)$$

$$k = \frac{\sqrt{2}}{10}$$

$$\therefore \frac{k \sin a}{k \cos a} = \frac{0.1}{0.1} = 1$$

$\tan a = 1, \; a = \pi/4$

$$x_{ss} = \frac{\sqrt{2}}{10} \sin\left(t + \pi/4\right)$$

The steady-state solution of the model is $\frac{\sqrt{2}}{10} \sin(t + \pi/4)$.