

Q.

The dynamic model of a body in motion performing damped forced vibration is

a Sin equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that  $t=0$ ,  $x=0.1$  and  $\frac{dx}{dt}=0$

- Using the auxiliary equation method, obtain the solution of the model information of an expression having  $x$  as a function of  $t$ .
- With the aid of matlab mfile program, plot the relationship between  $x$  and  $t$  for  $0 \leq t \leq 15$  time interval using a step size of 0.01 unit end.
- Write the steady state solution of the model in form of  $x = k \sin(\omega t - \alpha)$

Solution

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

In Auxiliary form

$$m^2 + 5m + 6 = 0$$

$$m^2 + 2m + 3m + 6 = 0$$

$$(m^2 + 2m)(3m + 6)$$

$$m(m+2) \cdot 3(m+2)$$

$$m = -2, m = -3$$

$$x = Ae^{-2t} + Be^{-3t}$$

$$PI = \cos t$$

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

Substituting back into equation

$$-C \cos t - D \sin t + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$-C \cos t + 6C \cos t - D \sin t + 6D \sin t - 5C \sin t + 5D \cos t = \cos t$$

$$5C \cos t + 5D \sin t - 5C \sin t + 5D \cos t = \cos t$$

$$5c \cos t + 5D \cos t + 5D \sin t - 5c \sin t = \cos t$$

$$5c + 5D = 1 \dots (i)$$

$$5D \sin t - 5c \sin t = 0$$

$$5D - 5c = 0 \dots (ii)$$

Using Simultaneous equation

$$5c + 5D = 1$$

$$-5c + 5D = 0$$

$$10D = 1$$

$$D = \frac{1}{10}$$

$$5c + 5\left(\frac{1}{10}\right) = 1$$

$$5c + \frac{1}{2} = 1$$

$$5c = 1 - \frac{1}{2}$$

$$5c = \frac{1}{2}$$

$$c = \frac{1}{10}$$

$$G.S = Ae^{-2t} + Be^{-3t} + \frac{1}{10} [\sin t + \cos t]$$

when  $t=0$ ,  $x=0.1$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10} [\sin(0) + \cos(0)]$$

$$0.1 = A + B + 0 + \frac{1}{10}$$

$$A + B = 0 \dots (i)$$

when  $t=0$ ,  $\frac{dx}{dt} = 0$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + 0.1(\cos t - \sin t)$$

$$0 = -2A - 3B + 0.1$$

$$-0.1 = -2A - 3B$$

Remember  $A + B = 0 \dots (ii)$

$$A = -B$$

$$-0.1 = -2(-B) - 3B$$

$$-0.1 = 2B - 3B$$

$$-0.1 = -B$$

$$B = 0.1$$

Knowing that  $A = -B$

$$A = -0.1$$

$$x = -0.1e^{-3t} + 0.1e^{-3t} + \frac{1}{10} [\sin t + \cos t] \text{ OR}$$

$$x = \frac{-1}{10} e^{-3t} + \frac{1}{10} e^{-3t} + \frac{1}{10} [\sin t + \cos t]$$

2) Command Window

Clear

clc

close all

Syms t

$$x = (\frac{1}{10} * \exp(-3*t) - (\frac{1}{10} * \exp(-3*t))) + (\frac{1}{10} (\sin t) + \cos t)$$

$$t = 0:0.01:1.5$$

$$xt = \text{subs}(x, t)$$

$$xt_n = \text{double}(xt)$$

Plot (t, x, t\_n)

x label ('t')

y label ('x')

grid on

grid minor

grid right

3. At steady state

$$\lim_{t \rightarrow \infty} x = \text{steady stat} = 0.1 \cos t + 0.1 \sin t$$

$$0.1 \cos t + 0.1 \sin t = K \sin(t + \alpha)$$

$$K \sin(t + \alpha) = K \sin \alpha + \cos \alpha + K \cos t \sin \alpha$$

NB Coefficient of  $\cos t = K \sin \alpha$

" " "  $\sin t = K \cos \alpha$

When Squaring both sides

$$K^2 \sin^2 \alpha + K^2 \cos^2 \alpha = 0.1^2 + 0.1^2$$

$$K^2 (\sin^2 \alpha + \cos^2 \alpha) = 0.02$$

$$K^2 = 0.02$$

$$K = \sqrt{0.02}$$

$$K = 0.144 = \sqrt{2}/10$$

$$\frac{K \sin \theta}{K \cos \theta} = \frac{0.1}{0.1} = 1$$

Remember that  $\sin / \cos = \tan$

$$\tan a = 1$$

$$\tan^{-1}(1) = a$$

$$a = 45^\circ \text{ or } \pi/4 \text{ radian}$$

Steady

$$= \frac{\sqrt{2}}{10} \sin(t + \pi/4)$$

