

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$X = CF + PI$$

for CF: $m^2 + 5m + 6 = 0$

$$m^2 + 3m + 2m + 6 = 0$$

$$m(m+3) + 2(m+3) = 0$$

$$(m+2)(m+3) = 0$$

$$m_1 = -2 \quad m_2 = -3$$

$$\therefore y = Ae^{-2t} + Bx e^{-3t}$$

for PI: $x = \cos t = C \cos t + D \sin t$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

$$\therefore -C \cos t - D \sin t + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$= -C \cos t - D \sin t + 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$= \cos t (-C + 5D + 6C) + \sin t (-D - 5C + 6D) = \cos t$$

$$= \cos t (5C + 5D) = \cos t \quad \sin t (-5C + 5D) = 0$$

$$5C + 5D = 1 \quad + \quad -5C + 5D = 0$$

$$0 + 10D = 1$$

$$D = \frac{1}{10}$$

$$5C + 1/5 (1/10) = 1$$

$$5C + \frac{1}{2} = 1$$

$$5C = 1 - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}$$

$$5C = \frac{1}{2}$$

$$C = \frac{1/2}{5} = \frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$$

$$\therefore X = \frac{1}{10} \cos t + \frac{1}{10} \sin t = \frac{1}{10} (\cos t + \sin t)$$

$$\therefore X = CF + PI$$

$$= Ae^{-2x} + Bx e^{-3x} + \frac{1}{10} (\cos t + \sin t)$$

when $t=0, x=0.1$

$$0.1 = Ae^{-2(0)} + Bx e^{-3(0)} + \frac{1}{10} (\cos(0) + \sin(0))$$

$$0.1 = A + 0 + \frac{1}{10} \times (1 + 0)$$

$$0.1 = A + \frac{1}{10} \times 1$$

$$0.1 = A + \frac{1}{10}$$

$$A = 0.1 + \frac{1}{10} = 0.1 + 0.1 = 0.2$$

$$A = 0.2 = \frac{2}{10}$$

0.

$$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10} (\cos t + \sin t)$$

When $t = 0, x = 0.1$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10} (\cos(0) + \sin(0))$$

$$0.1 = A + B + \frac{1}{10} (1 + 0)$$

$$0.1 = A + B + \frac{1}{10}$$

$$A + B = 0.1 - 0.1 = 0$$

$$A + B = 0 \quad \dots \quad 1$$

When $t = 0, \frac{dx}{dt} = 0$

$$\frac{dx}{dt} = -2Ae^{-2t} + 3Be^{-3t} + \frac{1}{10} (-\sin t + \cos t)$$

$$0 = -2Ae^{-2(0)} + 3Be^{-3(0)} + \frac{1}{10} (-\sin(0) + \cos(0))$$

$$0 = -2A - 3B + \frac{1}{10} (-0 + 1)$$

$$0 = -2A - 3B + \frac{1}{10} \quad \oplus$$

$$-2A - 3B = -\frac{1}{10}$$

$$2A + 3B = 0.1 \quad \dots \quad 2$$

$$A + B = 0 \quad \times 2$$

$$2A + 3B = 0.1 \quad \times 1$$

$$2A + 2B = 0$$

$$2A + 3B = 0.1$$

$$0 + -B = -0.1$$

$$B = 0.1$$

$$A + 0.1 = 0$$

$$A = -0.1$$

$$\therefore x = -0.1e^{-2t} + 0.1e^{-3t} + \frac{1}{10} (\cos t + \sin t)$$

$$x = \frac{1}{10} (e^{-3t} - e^{-2t}) + \frac{1}{10} (\cos t + \sin t)$$

$$x = \frac{1}{10} ((e^{-3t} - e^{-2t}) + (\cos t + \sin t))$$

$$x = K \sin(t + \phi)$$

$$0 = K \sin(0 + \phi)$$

$$0 = K \sin(0 + \pi)$$

MATLAB

Command Window

Clear

clc

Close all

sym t

$$x = 0.1 * \exp(-3 * t) - \exp(-2 * t) + 0.1 * (\cos(t) + \sin(t))$$

$$t = 0 : 0.01 : 15$$

plot(x, t)