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17/ENG05/039
MECHATRONICS ENGR.
ENG 381.

Assignment 1.

① The dynamic model of a body in motion performing damped force vibration is as eqn ①

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos(t)$$

Given that when $t = 0$, $x = 0.1$ $dx/dt = 0$

ⓐ Using the auxillary equation method, obtain the solution of the model in form of an expression having x as a function of t,

ⓑ With the aid of a MATLAB m-file program, plot the relationship between x and t for $0 \leq t \leq 15$ time unit using a step size of 0.01 unit and

ⓒ Write the steady state solution of the model in form of $x = 16 \sin(t+a)$.

Solution.

C.F = $m^2 + 5m + 6 = 0$

$(m+3)(m+2) = 0$

$m = -3$ $m = -2$

$x = Ae^{-3t} + Be^{-2t}$

$f(t) = \cos t$

$x = C\cdot\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cdot\sin t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$.

Subs.

$= -C\cdot\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$= \cos t(-C + 5D + 6C) + \sin t\, C - D - 5C + 6D = \cos t$

$= \cos t(5D + 5C) + \sin t(-5C + 5D) = \cos t.$

$5D + 5C = 1$

$5D + 5D - 5C = 0$

$10D = 1, \quad D = \frac{1}{10}$

to find C

$5\left(\frac{1}{10}\right) + 5C = 1$

$\frac{1}{2} + 5C = 1$

$C = \frac{1}{10}$

G.S $= x = Ae^{-3t} + Be^{-2t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

Subs. $(x = 0.1)$ $t = 0$

$0.1 = Ae^{-0} + Be^{-0} + \frac{1}{10}\cos 0 + \frac{1}{10}\sin 0$

$0.1 = A + B + 0.1$

$A + B = 0$ --- equ. (i)

$dx/dy = -3Ae^{-3t} - 2Be^{-2t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$

$0 = -3A - 2B + \frac{1}{10}$

$3A + 2B = \frac{1}{10}$ --- equ (ii)

$A + B = 0$ --- $\times 2$

$3A + 2B = 0.1$ --- $\times 1$

$2A + 2B = 0$

$3A + 2B = 0.1$

$-A = -0.1$

$A = 0.1$

To find B

$0.1 + B = 0$

$B = -\frac{1}{10}$ or -0.1

G.S $= -\frac{1}{10}e^{-3t} + \frac{1}{10}e^{-2t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

G.S $= \frac{1}{10}e^{-2t} - \frac{1}{10}e^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$= \frac{1}{10}(e^{-2t} - e^{-3t} + \cos t + \sin t)$

(iii) MATLAB myfile.

```
command window
clear
clc
close all
syms t
t = 0:0.01:15
x = 0.1*[exp(-2*t)-exp(-3*t)+cos(t)+sin(t)]
xn = subs(x)
plot(t, xn)
axis tight
grid on
grid minor.
```