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Department: ELECTRICAL/ELECTRONICS ENGINEERING
Course: ENG 381 (ENGINEERING MATHEMATICS)

ASSIGNMENT

1. The dynamic model of a body in motion performing damped force Vibrations is as in Eqn(i)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos(t)$$

Given that when $t=0$, $x=0.1$ $dx/dt = 0$

a. Using the Auxiliary Equation method, obtain the solution of the model in form of an expression having x as a function of t,

b. with the aid of a MATLAB mfile program plot the relationship between x and t for $0 \le t \le 15$ time unit using a Step Size of 0.01 unit, and

c. write the steady state solution of the model in form of $x = k\sin(t+a)$

Soln

$$CF: m^2+5m+6=0$$
$$(m+3)(m+2)=0$$
$$m_1 = -3 \quad m_2 = -2$$
$$x = Ae^{-3t} + Be^{-2t}$$
$$f(t) = \cos t$$
$$x = C\cos t + D\sin t$$
$$dx/dt = -C\sin t + D\cos t$$
$$d^2x/dt^2 = -C\cos t - D\cos t$$

Substituting.

$$= -C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$$
$$= \cos t(-C+5D+6C) + \sin t(-D+6D-5C) = \cos t$$
$$= \cos t(5D+5C) + \sin t(5D-5C) = \cos t$$

$$5D + 5C = 1$$
$$5D - 5C = 0$$
$$10D = 1$$
$$D = 1/10$$

to find C

$$5\left(\frac{1}{10}\right) + 5C = 1$$
$$\frac{1}{2} + 5C = 1$$
$$C = \frac{1}{10}$$

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G.S = $x = Ae^{-3t} + Be^{-2t} + 0.1\cos t + 0.1\sin t$

Subst $(x = 0.1)$, $t = 0$

$0.1 = Ae^{-0} + Be^{-0} + 0.1\cos(0) + 0.1\sin(0)$

$0.1 = A + B + 0.1$

$A + B = 0$ ——— eqn (1)

$dx/dt = -3Ae^{-3t} - 2Be^{-2t} - 0.1\sin t + 0.1\cos t$

$0 = -3A - 2B + 0.1$

$3A + 2B = 0.1$ ——— eqn (2)

$A + B = 0$ ——— $\times 2$

$3A + 2B = 0.1$ ——— $\times 1$

$2A + 2B = 0$

$3A + 2B = 0.1$

$-A = -0.1$

$A = 0.1$

to find B

$0.1 + B = 0$

$B = -0.1$

G.S = $0.1e^{-3t} - 0.1e^{-2t} + 0.1\cos t + 0.1\sin t$

$0.1\left(e^{-3t} - e^{-2t} + \cos t + \sin t\right)$

ii) MATLAB file

command window :

```
clear
clc
close all
t = 0:0.01:15
x = 0.1*[exp(-3*t)-exp(-2*t)+cos(t)+sin(t)]
Xn = subs(x)
plot(t, Xn)
axis tight
grid on
grid minor
```

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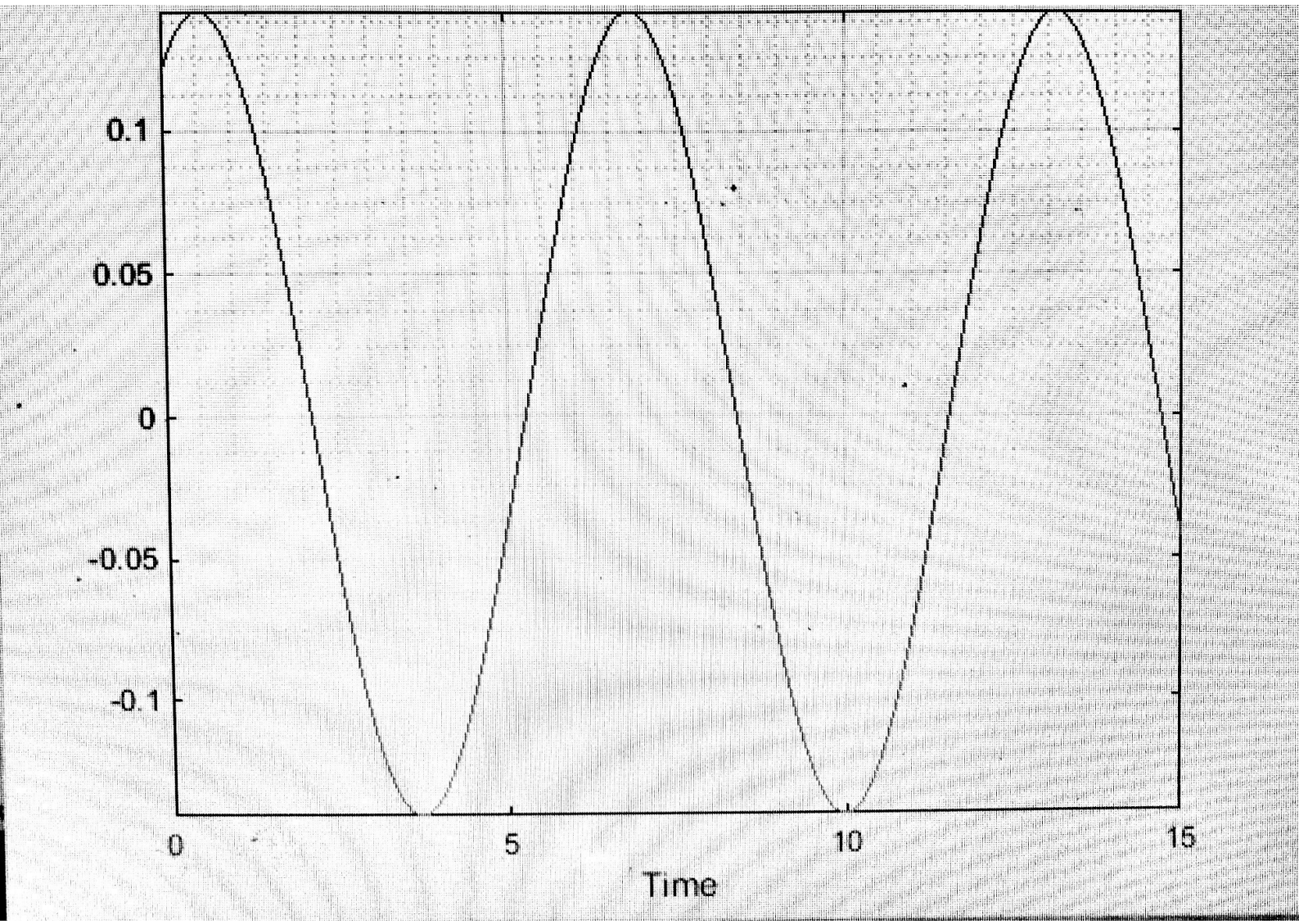

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c. $x_{ss} = x_{t\to\infty} = 0.1\cos t + 0.1\sin t = K\sin(t+\alpha)$

$x_{ss} = K\sin t\cos\alpha + K\cos t\sin\alpha$

$0.1 = K\sin\alpha,\ 0.1 = K\cos\alpha$

$K^2\sin^2\alpha + K^2\cos^2\alpha = \frac{1}{100} + \frac{1}{100}$

$K^2(\sin^2\alpha + \cos^2\alpha) = \frac{2}{100}$

$K^2 = \frac{2}{100}$ (since $\sin^2\alpha + \cos^2\alpha = 1$)

$K = \frac{\sqrt{2}}{100}$

$\frac{K\sin\alpha}{K\cos\alpha} = \frac{0.1}{0.1} = 1$

$\tan\alpha = 1,\ \alpha = \pi/4$

$x_{ss} = \frac{\sqrt{2}}{10}\sin\left(t + \pi/4\right)$

The steady-state solution of the model

$\frac{\sqrt{2}}{10}\sin\left(t + \frac{\pi}{4}\right)$

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