

Adeyokoko - Olagbo Samuel Charles

1815NG031006

Civil Engineering
ENGS 281 (Calculus)

$$4) \lim_{x \rightarrow 0} f(x) = \frac{a}{b} \quad \text{for } f(x) = \frac{\sin ax}{bx}$$

$$\lim_{x \rightarrow 0} \frac{\sin ax}{bx}$$

By direct substitution

$$\lim_{x \rightarrow 0} \frac{\sin ax}{bx} = \frac{\sin(a \cdot 0)}{b \cdot 0} = \frac{0}{0} = \text{undefined}$$

Apply L'Hopital's rule or substitute

$$\lim_{x \rightarrow 0} \frac{\sin ax}{bx} = \frac{\cos ax}{b}$$

$$\lim_{x \rightarrow 0} \frac{\cos ax}{b} = \frac{\cos(a \cdot 0)}{b} = \frac{1}{b} \quad (C_{30} = 1)$$

$$\frac{\sin ax}{bx} = \frac{a}{b}$$

$$f(x) = 5x - 21$$

$$d = 0.1 = \Delta d = 0.01$$

Let x_2 and x_1 be values of x such that:

$$x_2 - x_1 = d = 0.1 \quad \text{then } f(x_2) - f(x_1) = \epsilon$$

$$\epsilon = 5x_2 - 21 - [5x_1 - 21]$$

$$\epsilon = 5x_2 - 5x_1 - 21 + 21$$

$$\epsilon = 5(x_2 - x_1)$$

$$\epsilon = 5 \times d$$

$$\epsilon = 5 \times 0.1$$

$$\epsilon = 0.5 = 5 \times 0.1 = 0.05$$

x	$f(x)$
5.11	8.5
5.11	8.55
5.12	8.60
5.15	8.66
5.14	8.70
5.15	8.75
5.16	8.80
5.12	8.85
5.18	8.90
5.19	8.95

x	$f(x)$
6.1	9.5
6.09	9.45
6.08	9.40
6.06	9.35
6.05	9.30
6.04	9.25
6.03	9.20
6.02	9.15
6.01	9.10
	9.05

$f(x) = (25 - x^2)^{1/2}$ (C) Interval $[-5, 5]$

A function is continuous if

$\lim_{x \rightarrow a} f(x) = \text{def. val.}$

$x \rightarrow a$
 $\lim_{x \rightarrow a} f(x) = f(a)$

$x \rightarrow 5$
 $f(x) = \text{def. val.}$

Between the interval $[-5, 5]$

$$\begin{aligned}
 \text{at } x = 5 \\
 f(-5) &= 25 - (-5)^2)^{1/2} \\
 &= (25 - 25)^{1/2} \\
 &= 0 \text{ (defined val.)}
 \end{aligned}$$

$$a + x = 0$$

$$f(0) = (25 - 0^2)^{1/2}$$

$$f(0) = (25)^{1/2}$$

$$f(0) = 5 \text{ (def. val.)}$$

$$a + x = 5$$

$$f(5) = (25 - (5^2))^{1/2}$$

$$f(5) = 0$$

$$\lim_{x \rightarrow -5} [25 - x^2]^{1/2} = [25 - (-5)^2]^{1/2}$$

$$= 0 \quad \left\{ \begin{array}{l} \text{cont. def.} \\ \text{cont. def.} \end{array} \right.$$

$$\lim_{x \rightarrow 0} [25 - x^2]^{1/2} = [25 - 0^2]^{1/2}$$

$$= (25)^{1/2}$$

$$\lim_{x \rightarrow 0} (25 - x^2)^{1/2} = 5$$

$$\lim (25 - x^2)^{1/2} = (25 - 5^2)^{1/2}$$

$$\lim_{x \rightarrow 5} (25 - x^2)^{1/2} = 0$$

At $x = -5$, $x = 0$ and $x = 5$

$$\lim_{x \rightarrow -5} f(x) = f(-5), \quad f(-5) = \text{defined}$$

$$\lim_{x \rightarrow 0} f(x) = f(0), \quad f(0) = \text{defined}$$

$$\lim_{x \rightarrow 5} f(x) = f(5), \quad f(5) = \text{defined}$$

Therefore $f(x) = [25 - x^2]^{1/2}$ is continuous on the interval $[-5, 5]$