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Chemical Engineering

ENG 381 Assignment

Solution

$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = Cost$

Homogenous Equation - $d^2x/dt^2 + 5dx/dt + 6x = 0$

$m^2 + 5m + 6 = 0$

$m^2 + 2m + 3m + 6 = 0$

$(m^2 + 2m) + (3m + 6) = 0$

$m(m+2) + 3(m+2) = 0$

$(m+3)(m+2) = 0$

$m_1 = -3 \quad m_2 = -2 \quad // m_1 = -2 \quad m_2 = -3$

$x = Ae^{mx} + Be^{mx}$

$x = Ae^{-2t} + Be^{-3t} \Rightarrow C.F$

$P.I \Rightarrow f(x) = Cost$

$x = Ccost + Dsint$

$dx/dt = -Csint + Dcost$

$d^2x/dt^2 = -Ccost - Dsint$

$\Rightarrow \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

$= (-C\cos t - D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$6C\cos t - C\cos t + 5D\cos t + 6D\sin t - D\sin t - 5C\sin t = \cos t$

$5C\cos t + 5D\cos t + 5D\sin t - 5C\sin t = \cos t$

$\cos t(5C + 5D) + \sin t(5D - 5C) = \cos t(1) + \sin t(0)$

$5C + 5D = 1 \quad - - - (i)$

$5D - 5C = 0 \quad - - - (ii)$

From eqn (ii)

$5D = 5C \quad - - - (iii)$

Substitute eqn (iii) into eqn (i)

$5C + 5C = 1$

$10C = 1$

$C = 1/10$

From eqn (iii)

$5D = 5C$

$5D = 5(1/10)$

$5D = 1/2$

$D = 1/2 \div 5 \Rightarrow 1/2 \times \frac{1}{5} \Rightarrow 1/10$

$P.I \Rightarrow x = C\cos t + D\sin t$

$x = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$x = \frac{1}{10}(\cos t + \sin t)$

$G.S = C.F + P.I$

$G.S = Ae^{-2t} + Be^{-3t} + \frac{1}{10}(\cos t + \sin t)$

When $t = 0$, $x = 0.1$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}(\cos(0) + \sin(0))$

$0.1 = A + B + \frac{1}{10}$

$A + B = 0.1 - \frac{1}{10}$

$A + B = 0 \quad ---(i)$

When $t = 0$, $dx/dt = 0$

$dx/dt = -2Ae^{-2t} - 3Be^{-3t} + \frac{1}{10}(-\sin t + \cos t)$

$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} + \frac{1}{10}(-\sin(0) + \cos(0))$

$0 = -2A - 3B + 0.1$

$2A + 3B = 0.1$

From eqn (i)

$A = -B$

$-2B + 3B = 0.1$

$B = 0.1$

Since, $A = -B$

$\therefore A = -0.1$

* $x = -0.1e^{-2t} + 0.1e^{-3t} + 0.1(\cos t + \sin t)$

2 Command window

```
clear
clc
close all
syms t
x = (0.1*exp[-3*t]) - (0.1*exp[-2*t]) + 0.1*cost + 0.1*sint
t = 0:0.01:15
xn = subs(x,t)
xnn = Double(xn)
Plot(t, xnn)
x Label ('t')
y Label ('x')
axis tight
grid on
grid minor
```

At steady state

$f(x) \Rightarrow x = 0.1\cos t + 0.1\sin t$

$f(x) = 0.1\cos t$ At steady state

$\lim_{t \to \infty} x$ = $x = 0.1\cos t + 0.1\sin t$ $t \to \infty$ = $x_{steady\ state} = 0.1\cos t + 0.1\sin t$

Steady state

$0.1\cos t + 0.1\sin t = K\sin(t + a)$

$K\sin(t+a) = K\sin t\cos a + K\cos t\sin a$

=> N.B:-

Coefficient of $\cos t = K\sin a$

" " $\sin t = K\cos a$

Square both sides

$K^2(\sin^2 a + \cos^2 a) = 0.1^2 + 0.1^2$

$K^2(\sin^2 a + \cos^2 a) = 0.02$

From trig functions $\sin^2\theta + \cos^2\theta = 1$

$K^2 = 0.02$

$K = \sqrt{2}/10$

$\frac{K\sin a}{K\cos a} = \frac{0.1}{0.1} = 1$

Recall, $\sin\theta/\cos\theta = \tan\theta$

$\tan a = 1$

$a = \tan^{-1}(1)$

$a = 45°$ // $\pi/4$ rad

=> Steady state $= \sqrt{2}/10\sin(t + \pi/4)$