AGWANIRU ROSEMARY
17/ENG01/003
CHEMICAL ENGINEERING
CHE 381: ENGINEERING MATHS III

SOLUTION TO ASSIGNMENT

1) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

Assuming $f(t) = 0$

$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$

In auxillary form

$m^2 + 5m + 6 = 0$

$m^2 + 2m + 3m + 6 = 0$

$m(m+2) 3(m+2) = 0$

$(m+3)(m+2) = 0$

$m_1 = -3, \quad m_2 = -2$

$x = Ae^{m_1 t} + Be^{m_2 t}$

$x = Ae^{-2t} + Be^{-3t}$

P.I =

$x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

Substituting back into equations

$-(\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t)$

$= \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$\cos t(-C + 5D + 6C) + \sin t(-D - 5C + 6D) = \cos t$

$\cos t(5C + 5D) + \sin t(5D - 5C) = \cos t$

$\cos t(5C + 5D) = 1\cos t$

$\sin t(5D - 5C) = 0 \sin t$

$5C + 5D = 1 \quad \text{---} (i)$

$5D - 5C = 0 \quad \text{---} (ii)$

Using simultaneous equation

$5C + 5D = 1$

$-5C + 5D = 0$

$+10D = 1$

$\therefore D = \frac{1}{10}$

$5C + 5(1/10) = 1$

$5C + 1/2 = 1$

$5C = 1 - 1/2$

$C = 1/2 \times 1/5$

$C = 1/10$

$\therefore P.I = 1/10(\cos t + \sin t)$

$G.S = Ae^{-2t} + Be^{-3t} + 1/10[\cos t + \sin t)$

when $t = 0, x = 0.1$

$0.1 = Ae^{-3(0)} + Be^{-2(0)} + 1/10[\cos(0) + \sin(0)]$

$0.1 = A + B + 1/10$

$0.1 - 1/10 = A + B$

$0 = A + B \quad \text{---} (i)$

$A = -B$

when $t = 0 \quad \frac{dx}{dt} = 0$

$\frac{dx}{dt} = -2Ae^{-3t} + 3Be^{-2t} + 1/10[-\sin t + \cos t)$

$0 = -2Ae^{-3(0)} - 3Be^{-2(0)} + 1/10[-\sin(0) + \cos(0)]$

$0 = -2A - 3B + 1/10$

$-0.1 = -2A - 3B$

$-0.1 = -(2A + 3B)$

$0.1 = 2A + 3B \quad \text{---} (ii)$

recall from eqn(i) $A = -B$

$0.1 = 2(-B) + 3B$

$0.1 = -2B + 3B$

$0.1 = B$

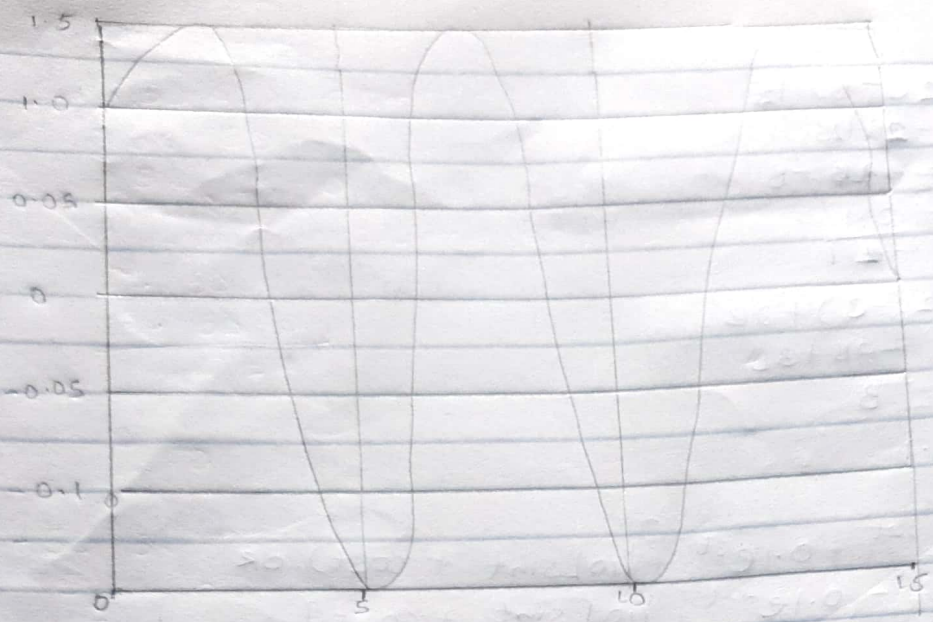
$\therefore A = -0.1$

$\therefore x = -0.1e^{-2t} + 0.1e^{3t} + 1/10[\sin t + \cos t)$ OR

$x = 0.1e^{3t} - 0.1e^{-2t} + 1/10[\sin t + \cos t]$

2) Command Window

```
clear
clc
close all
syms t
x = (1/10 * exp(-2*t) - (1/10 * exp(-3*t)) + (1/10(sin t) + cos(t))
t = 0 : 0.01 :
xt = subs (x, t)
xtn = double (xt)
plot ( t, x, n)
xlabel ('t')
ylabel ('x')
grid on
grid minor
grid
```



3 At steady state

$x_{t\to\infty} = x_{\text{steady state}} = 0.1\cos t + 0.1\sin t$

$0.1\cos t + 0.1\sin t = k\sin(t+a)$

$k\sin(t+a) = k\sin t\cos a + k\cos t\sin a$

co-efficent of $\cos t = k\sin a$

co-efficient of $\sin t = k\cos a$

When squaring both sides

$k^2\sin^2 a + k^2\cos^2 a = 0.1^2 + 0.1^2$

$k^2(\sin^2 a + \cos^2 a) = 0.02$

$k^2 = 0.02$

$k = \sqrt{0.02} = 0.144 = \sqrt{2}/10$

$\frac{1\cdot\sin a}{1\cos a} = \frac{0.1}{0.1} = 1$

recall $\sin/\cos = \tan$

$\tan a = 1$

$\tan^{-1}(1) = a$

$a = 45^\circ$ or $\pi/4$ radian

Steady state

$= \frac{\sqrt{2}}{10}(\sin(t+\pi/a))$